Precalculus for Team-Based Inquiry Learning

2024 Development Edition

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Website: Precalculus for Team-Based Inquiry Learning[1]

[1] teambasedinquirylearning.github.io/precalculus/

[2] github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Equations, Inequalities, and Applications (EQ)

Objectives

How do we find solutions of equations?
By the end of this chapter, you should be able to...

1. Solve linear equations in one variable. Solve linear inequalities in one variable and express the solution graphically and using interval notation.

2. Solve application problems involving linear equations.

3. Given two points, determine the distance between them and the midpoint of the line segment connecting them.

4. Solve linear equations involving an absolute value. Solve linear inequalities involving absolute values and express the answers graphically and using interval notation.

5. Solve quadratic equations using factoring, the square root property, completing the square, and the quadratic formula and express these answers in exact form.

6. Solve rational equations.

7. Solve quadratic inequalities and express the solution graphically and with interval notation. Solve rational inequalities and express the solution graphically and using interval notation.

1.1 Linear Equations and Inequalities (EQ1)

Objectives

- Solve linear equations in one variable. Solve linear inequalities in one variable and express the solution graphically and using interval notation.

Remark 1.1.1 Recall that when solving a linear equation, you use addition, subtraction, multiplication and division to isolate the variable.

Activity 1.1.2 Solve the linear equations.

(a) $3x - 8 = 5x + 2$

A. $x = 2$

B. $x = 5$

C. $x = -5$

D. $x = -2$

(b) $5(3x - 4) = 2x - (x + 3)$

A. $x = \dfrac{17}{14}$

B. $x = \dfrac{14}{17}$

C. $x = \dfrac{23}{14}$

D. $x = \dfrac{14}{23}$

Activity 1.1.3 Solve the linear equation.

$$\frac{2}{3}x - 8 = \frac{5x+1}{6}$$

(a) Which equation is equivalent to $\frac{2}{3}x - 8 = \frac{5x+1}{6}$ but does not contain any fractions?

A. $12x - 48 = 15x + 3$

B. $3x - 24 = 10x + 2$

C. $4x - 8 = 5x + 1$

D. $4x - 48 = 5x + 1$

(b) Use the simplified equation from part (a) to solve $\frac{2}{3}x - 8 = \frac{5x+1}{6}$.

A. $x = -17$

B. $x = -\dfrac{26}{7}$

C. $x = -9$

D. $x = -49$

Activity 1.1.4 It is not always the case that a linear equation has exactly one solution. Consider the following linear equations which appear similar, but their solutions are very different.

(a) Which of these equations has one unique solution?

A. $4(x-2) = 4x+6$

B. $4(x-1) = 4x-4$

C. $4(x-1) = x+4$

(b) Which of these equations has no solutions?

A. $4(x-2) = 4x+6$

B. $4(x-1) = 4x-4$

C. $4(x-1) = x+4$

(c) Which of these equations has many solutions?

A. $4(x-2) = 4x+6$

B. $4(x-1) = 4x-4$

C. $4(x-1) = x+4$

(d) What happens to the x variable when a linear equation has no solution or many solutions?

Definition 1.1.5 A linear equation with one unique solution is a **conditional equation.** A linear equation that is true for all values of the variable is an **identity equation**. A linear equation with no solutions is an **inconsistent equation.** $\Diamond$

Activity 1.1.6 An inequality is a relationship between two values that are not equal.

(a) What is the solution to the linear equation $3x - 1 = 5$?

(b) Which of these values is a solution of the inequality $3x - 1 \geq 5$?

A. $x = 0$

B. $x = 2$

C. $x = 4$

D. $x = 10$

(c) Express the solution of the inequality $3x - 1 \geq 5$ in interval notation.

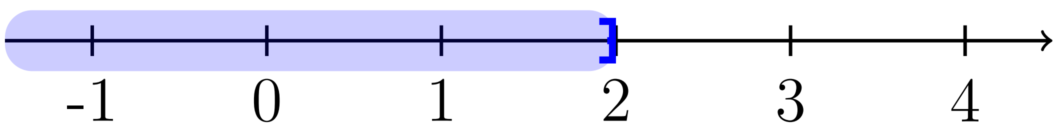
A. $(-\infty, 2]$

B. $(-\infty, 2)$

C. $(2, \infty)$

D. $[2, \infty)$

(d) Draw the solution to the inequality on a number line.



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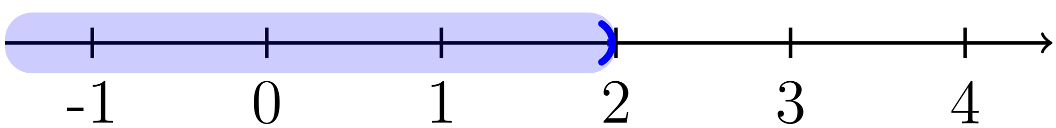

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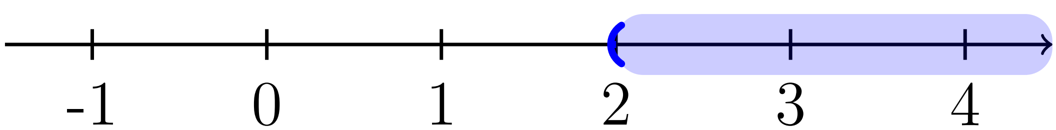

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Activity 1.1.7 Let's consider what happens to the inequality when the variable has a negative coefficient.

(a) Which of these values is a solution of the inequality $-x < 8$?

A. $x = -10$

B. $x = -8$

C. $x = 4$

D. $x = 10$

(b) Solve the linear inequality $-x < 8$. How does your solution compare to the values chosen in part (a)?

(c) Expression the solution of the inequality $-x < 8$ in interval notation.

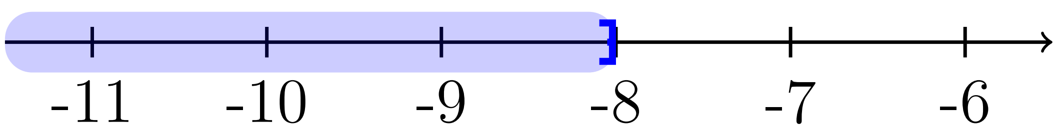
A. $(-\infty, -8]$

B. $(-\infty, -8)$

C. $(-8, \infty)$

D. $[-8, \infty)$

(d) Draw the solution to the inequality on a number line.



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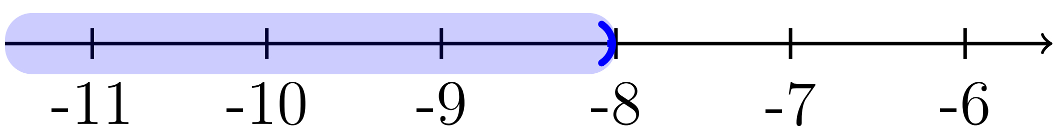

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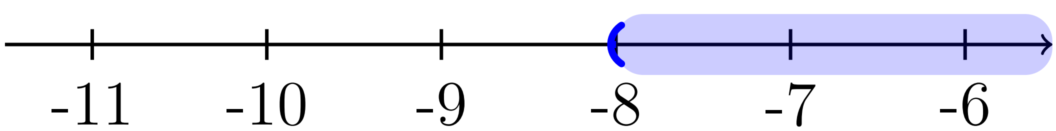

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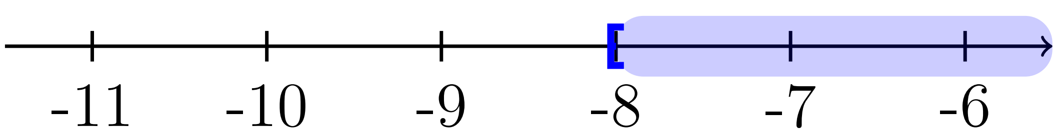

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Remark 1.1.8 You can treat solving linear inequalities, just like solving an equation. The one exception is when you multiply or divide by a negative value, reverse the inequality symbol.

Activity 1.1.9 Solve the following inequalities. Express your solution in interval notation and graphically on a number line.

(a) $-3x - 1 \leq 5$

(b) $3(x + 4) > 2x - 1$

(c) $-\frac{1}{2}x \geq -\frac{3}{4} + \frac{5}{4}x$

Definition 1.1.10 A **compound inequality** includes multiple inequalities in one statement. $\Diamond$

Activity 1.1.11 Consider the statement $3 \leq x < 8$. This really means that $3 \leq x$ and $x < 8$.

(a) Which of the following inequalities are equivalent to the compound inequality $3 \leq 2x - 3 < 8$?

A. $3 \leq 2x - 3$

B. $3 \geq 2x - 3$

C. $2x - 3 < 8$

D. $2x - 3 > 8$

(b) Solve the inequality $3 \leq 2x - 3$.

A. $x \leq 0$

B. $x \geq 0$

C. $x \leq 3$

D. $x \geq 3$

(c) Solve the inequality $2x - 3 < 8$.

A. $x > \frac{11}{2}$

B. $x < \frac{11}{2}$

C. $x > \frac{5}{2}$

D. $x < \frac{5}{2}$

(d) Which compound inequality describes how the two solutions overlap?

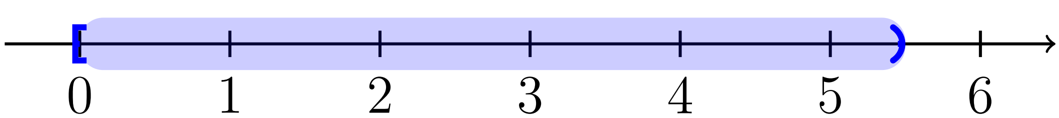
A. $0 \leq x < \frac{11}{2}$

B. $0 \leq x < \frac{5}{2}$

C. $\frac{5}{2} < x \leq 3$

D. $3 \leq x < \frac{11}{2}$

(e) Draw the solution to the inequality on a number line.



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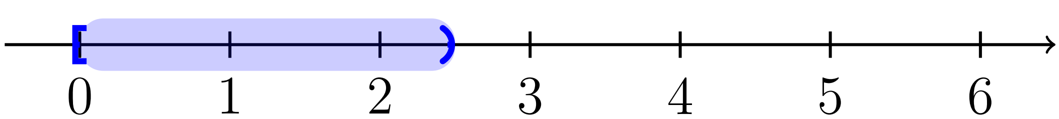

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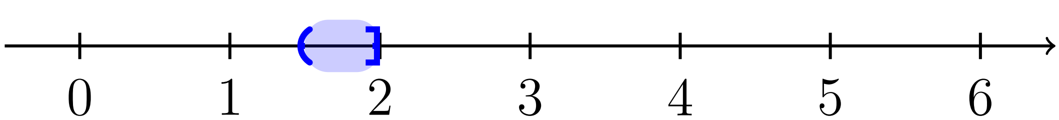

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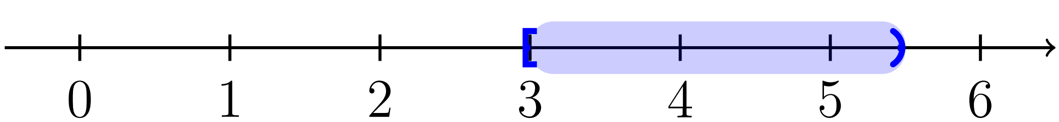


D.

Remark 1.1.12 Solving a compound linear inequality, uses the same methods as a single linear inequality ensuring that you perform the same operations on all three parts. Alternatively, you can break the compound inquality up into two and solve separately.

Activity 1.1.13 Solve the following inequalities. Express your solution in interval notation and graphically on a number line.

(a) $8 < -3x - 1 \leq 11$

(b) $-6 \leq \frac{x-12}{4} < -2$

1.2 Applications of Linear Equations (EQ2)

Objectives

- Solve application problems involving linear equations.

Applications of Linear Equations (EQ2)

Observation 1.2.1 Linear equations can be used to solve many types of real-world applications. We'll investigate some of those in this section.

Remark 1.2.2 Distance, rate, and time problems are a standard example of an application of a linear equation. For these, it's important to remember that

$$d = rt$$

where d is distance, r is the rate (or speed), and t is time.

Often we will have more than one moving object, so it is helpful to denote which object's distance, rate, or time we are referring to. One way we can do this is by using a subscript. For example, if we are describing an eastbound train (as we will in the first example), it may be helpful to denote its distance, rate, and time as d_E, r_E, and t_E respectively. Notice that the subscript E is a label reminding us that we are referring to the eastbound train.

Activity 1.2.3 Two trains leave a station at the same time. One is heading east at a speed of 75 mph, while the other is heading west at a speed of 85 mph. After how long will the trains be 400 miles apart?

(a) How fast is each train traveling?

A. $r_E = 85$ mph, $r_W = 75$ mph

B. $r_E = 75$ mph, $r_W = 85$ mph

C. $r_E = 400$ mph, $r_W = 400$ mph

D. $r_E = 75$ mph, $r_W = 400$ mph

E. $r_E = 400$ mph, $r_W = 85$ mph

(b) Which of the statements describes how the times of the eastbound and westbound train are related?

A. The eastbound train is slower than the westbound train, so $75 + t_E = 85 + t_W$.

B. The eastbound train left an hour before the westbound train, so if we let $t_E = t$, then $t_W = t - 1$.

C. Both trains have been traveling the same amount of time, so $t_E = t_W$. Since they are the same, we can just call them both t.

D. We don't know how the times relate to each other, so we must denote them separately as t_E and t_W.

E. Since the trains are traveling at different speeds, we need the proportion $\frac{r_E}{r_W} = \frac{t_E}{t_W}$.

(c) Fill in the following table using the information you've just determined about the trains' rates and times since they left the station. Some values are there to help you get started.

	rate	**time**	**distance from station**
eastbound train			$75t$
westbound train		t	

(d) At the moment in question, the trains are 400 miles apart. How does that total distance relate to the distance each train has traveled?

A. The 400 miles is irrelevant. They've been traveling the same amount of time so they must be the same distance away from the station. That tells us $d_E = d_W$.

B. The 400 miles is the difference between the distance each train traveled, so $d_E - d_W = 400$.

C. The 400 miles represents the sum of the distances that each train has traveled, so $d_E + d_W = 400$.

D. The 400 miles is the product of the distance each train traveled, so $(d_E)(d_W) = 400$.

(e) Now plug in the expressions from your table for d_E and d_w. What equation do you get?

A. $75t = 85t$

B. $75t - 85t = 400$

C. $75t + 85t = 400$

D. $(75t)(85t) = 400$

(f) Notice that we now have a linear equation in one variable, t. Solve for t, and put that answer in context of the problem.

A. The trains are 400 miles apart after 2 hours.

B. The trains are 400 miles apart after 2.5 hours.

C. The trains are 400 miles apart after 3 hours.

D. The trains are 400 miles apart after 3.5 hours.

E. The trains are 400 miles apart after 4 hours.

Remark 1.2.4 In Activity 1.2.3 we examined the motion of two objects moving at the same time in opposite directions. In Activity 1.2.5 we will examine a different perspective, but still apply $d = rt$ to solve.

Activity 1.2.5 Jalen needs groceries, so decides to ride his bike to the store. It takes him half an hour to get there. After finishing his shopping, he sees his friend Alex who offers him a ride home. He takes the same route home as he did to the store, but this time it only takes one-fifth of an hour. If his average speed was 18 mph faster on the way home, how far away does Jalen live from the grocery store?

We'll use the subscript b to refer to variables relating to Jalen's trip to the store while riding his *b*ike and the subscript c to refer to variables relating to Jalen's trip home while riding in his friend's *c*ar.

(a) How long does his bike trip from home to the store and his car trip from the store back home take?

A. $t_b = 18$ hours, $t_c = 18$ hours

B. $t_b = \frac{1}{5}$ of an hour, $t_c = \frac{1}{2}$ of an hour

C. $t_b = \frac{1}{2}$ of an hour, $t_c = \frac{1}{5}$ of an hour

D. $t_b = 2$ hours, $t_c = 5$ hours

E. $t_b = 5$ hours, $t_c = 2$ hours

(b) Which of the statements describes how the speed (rate) of the bike trip and the car trip are related?

A. Both the trip to the store and the trip home covered the same distance, so $r_b = r_c$. Since they are the same, we can just call them both r.

B. We don't know how the two rates relate to each other, so cannot write an equation comparing them and must leave them as separate variables r_b and r_c.

C. Jalen's rate on the trip home in the car was 18 mph faster than his trip to the store on his bike, so if we let $r_b = r$, then $r_c = r - 18$.

D. Jalen's rate on the trip home in the car was 18 mph faster than his trip to the store on his bike, so if we let $r_b = r$, then $r_c = r + 18$.

(c) Fill in the following table using the information you've just determined about the Jalen's rates and times on each leg of his grocery store trip. Then fill in the distance column based on how distance relates to rate and time in each case.

	rate	**time**	**distance covered**
bike trip (to the store)			
car trip (going back home)			

(d) Our goal is to figure out how far away Jalen lives from the store. To help us get there, write an equation relating d_b and d_c.

A. The distance he traveled by bike is the same as the distance he traveled by car, so $d_b = d_c$

B. The distance he traveled by bike took longer than the distance he traveled by car, so $d_b + \frac{1}{2} = d_c + \frac{1}{5}$

C. The distance, d, between his house and the grocery store is sum of the distance he traveled on his bike and the distance he traveled in the car, so $d_b + d_c = d$.

D. The distance, d, between his house and the grocery store is sum of the difference he traveled on his bike and the distance he traveled in the car, so $d_b - d_c = d$.

(e) Now plug in the expressions from your table for d_b and d_c into the equation you just found. Notice that it is a linear equation in one variable, r. Solve for r.

(f) Our goal was to determine the distance between Jalen's house and the grocery store. Solving for r did not tell us that distance, but it did get us one step closer. Use that value to help you determine the distance between his house and the store, and write your answer using the context of the problem. (Hint: can you find an expression involving r that we made that represents that distance?)

A. The grocery store is 6 miles away from Jalen's house.

B. The grocery store is 8 miles away from Jalen's house.

C. The grocery store is 10 miles away from Jalen's house.

D. The grocery store is 12 miles away from Jalen's house.

E. The grocery store is 14 miles away from Jalen's house.

Remark 1.2.6 Another type of application of linear equations is called a mixture problem. In these we will mix together two things, like two types of candy in a candy store or two solutions of different concentrations of alcohol.

Activity 1.2.7 Ammie's favorite snack to share with friends is candy salad, which is a mixture of different types of candy. Today she chooses to mix Nerds Gummy Clusters, which cost \$8.38 per pound, and Starburst Jelly Beans, which cost \$7.16 per pound. If she makes seven pounds of candy salad and spends a total of \$55.61, how many pounds of each candy did she buy?

(a) There are two "totals" in this situation: the total weight (in pounds) of candy Ammie bought and the total amount of money (in dollars) Ammie spent. Let's begin with the total weight. If we let N represent the pounds of Nerds Gummy Clusters and S represent the pounds of Starburst Jelly Beans, which of the following equations can represent the total weight?

A. $N - S = 7$

B. $NS = 7$

C. $N + S = 7$

D. $\dfrac{N}{S} = 7$

(b) Which expressions represent the amount she spent on each candy? Again, we will let N represent the pounds of Nerds Gummy Clusters and S represent the pounds of Starburst Jelly Beans.

A. N spent on Nerds Gummy Clusters; S spent on Starburst Jelly Beans

B. $8.38N$ spent on Nerds Gummy Clusters; $7.16S$ spent on Starburst Jelly Beans

C. $8.38 + N$ spent on Nerds Gummy Clusters; $7.16 + S$ spent on Starburst Jelly Beans

D. $8.38 - N$ spent on Nerds Gummy Clusters; $7.16 - S$ spent on Starburst Jelly Beans

(c) Now we focus on the total cost. Which of the following equations can represent the total amount she spent?

A. $N + S = 55.61$

B. $8.38N + 7.16S = 55.61$

C. $8.38 + N + 7.16 + S = 55.61$

D. $8.38 - N + 7.16 - S = 55.61$

(d) We are almost ready to solve, but we have two variables in our weight equation and our cost equation. We will get the cost equation to one variable by using the weight equation as a substitution. Which of the following is a way to express one variable in terms of the other? (Hint: More than one answer may be correct here!)

A. If N is the total weight of the Nerds Gummy Clusters, then $7-N$ could represent the weight of the Starburst Jelly Beans.

B. If N is the total weight of the Nerds Gummy Clusters, then $7+N$ could represent the weight of the Starburst Jelly Beans.

C. If S is the total weight of the Starburst Jelly Beans, then $7 - S$ could represent the weight of the Nerds Gummy Clusters.

D. If S is the total weight of the Starburst Jelly Beans, then $7 + S$ could represent the weight of the Nerds Gummy Clusters.

(e) Plug your expressions in to the total cost equation. (Hint: More than one of these may be correct!)

A. $8.38N + 7.16(7 - N) = 55.61$

B. $8.38S + 7.16(7 - S) = 55.61$

C. $8.38(7 - N) + 7.16N = 55.61$

D. $8.38(7 - S) + 7.16S = 55.61$

(f) Now solve for N and S, and put your answer in the context of the problem.

A. Ammie bought 2.5 lbs of Nerds Gummy Clusters and 4.5 lbs of Starburst Jelly Beans.

B. Ammie bought 3.5 lbs of Nerds Gummy Clusters and 3.5 lbs of Starburst Jelly Beans.

C. Ammie bought 4.5 lbs of Nerds Gummy Clusters and 2.5 lbs of Starburst Jelly Beans.

D. Ammie bought 5.5 lbs of Nerds Gummy Clusters and 1.5 lbs of Starburst Jelly Beans.

Activity 1.2.8 A chemist needs to mix two solutions to create a mixture consisting of 30% alcohol. She uses 20 liters of the first solution, which has a concentration of 21% alcohol. How many liters of the second solution (that is 45% alcohol) should she add to the first solution to create the mixture that is 30% alcohol?

1.3 Distance and Midpoint (EQ3)

Objectives

- Given two points, determine the distance between them and the midpoint of the line segment connecting them.

Activity 1.3.1 The points A and B are shown in the graph below. Use the graph to answer the following questions:

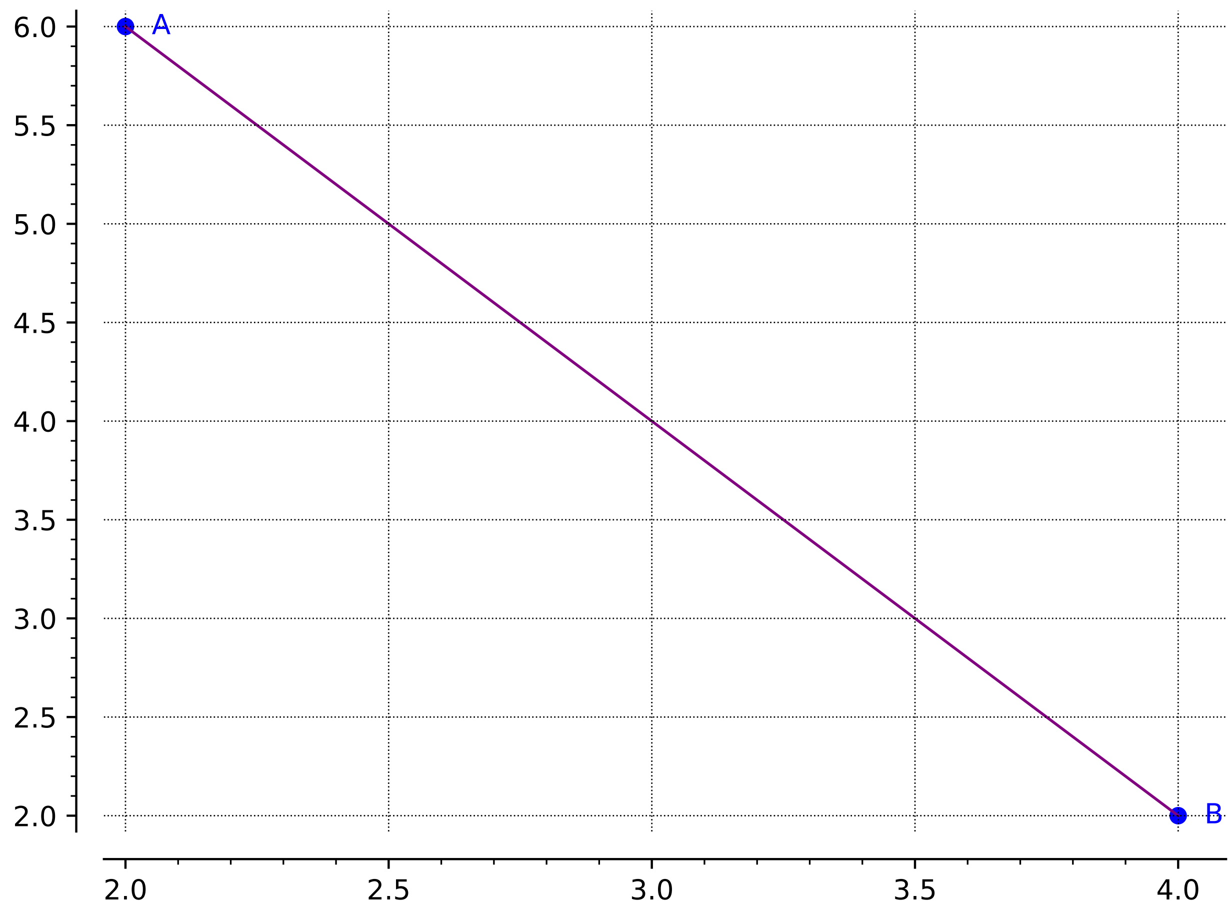


Figure 1.3.2

(a) Draw a right triangle so that the hypotenuse is the line segment between points A and B. Label the third point of the triangle C.

(b) Find the lengths of line segments AC and BC.

(c) Now that you know the lengths of AC and BC, how can you find the length of AB? Find the length of AB.

Remark 1.3.3 Using the **Pythagorean Theorem** ($a^2 + b^2 = c^2$) can be helpful in finding the distance of a line segment (as long as you create a right triangle!).

Activity 1.3.4 Suppose you are given two points (x_1, y_1) and (x_2, y_2). Let's investigate how to find the length of the line segment that connects these two points!

(a) Draw a sketch of a right triangle so that the hypotenuse is the line segment between the two points.

(b) Find the lengths of the legs of the right triangle.

(c) Find the length of the line segment that connects the two original points.

Definition 1.3.5 The distance, d, between two points, (x_1, y_1) and (x_2, y_2), can be found by using the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Notice that the distance formula is an application of the Pythagorean Theorem! $\diamondsuit$

Activity 1.3.6 Apply Definition 1.3.5 to calculate the distance between the given points.

(a) What is the distance between $(4, 6)$ and $(9, 15)$?

A. 10.2

B. 10.3

C. $\sqrt{106}$

D. $\sqrt{56}$

(b) What is the distance between $(-2, 5)$ and $(-7, -1)$?

A. $\sqrt{11}$

B. 7.8

C. 3.3

D. $\sqrt{61}$

(c) Suppose the line segment AB has one endpoint, A, at the origin. For which coordinate of B would make the line segment AB the longest?

A. $(3, 7)$

B. $(2, -8)$

C. $(-6, 4)$

D. $(-5, -5)$

Remark 1.3.7 Notice in Activity 1.3.6, you can give a distance in either exact form (leaving it with a square root) or as an approximation (as a decimal). Make sure you can give either form as sometimes one form is more useful than another!

Remark 1.3.8 A **midpoint** refers to the point that is located in the middle of a line segment. In other words, the midpoint is the point that is halfway between the two endpoints of a given line segment.

Activity 1.3.9 Two line segments are shown in the graph below. Use the graph to answer the following questions:

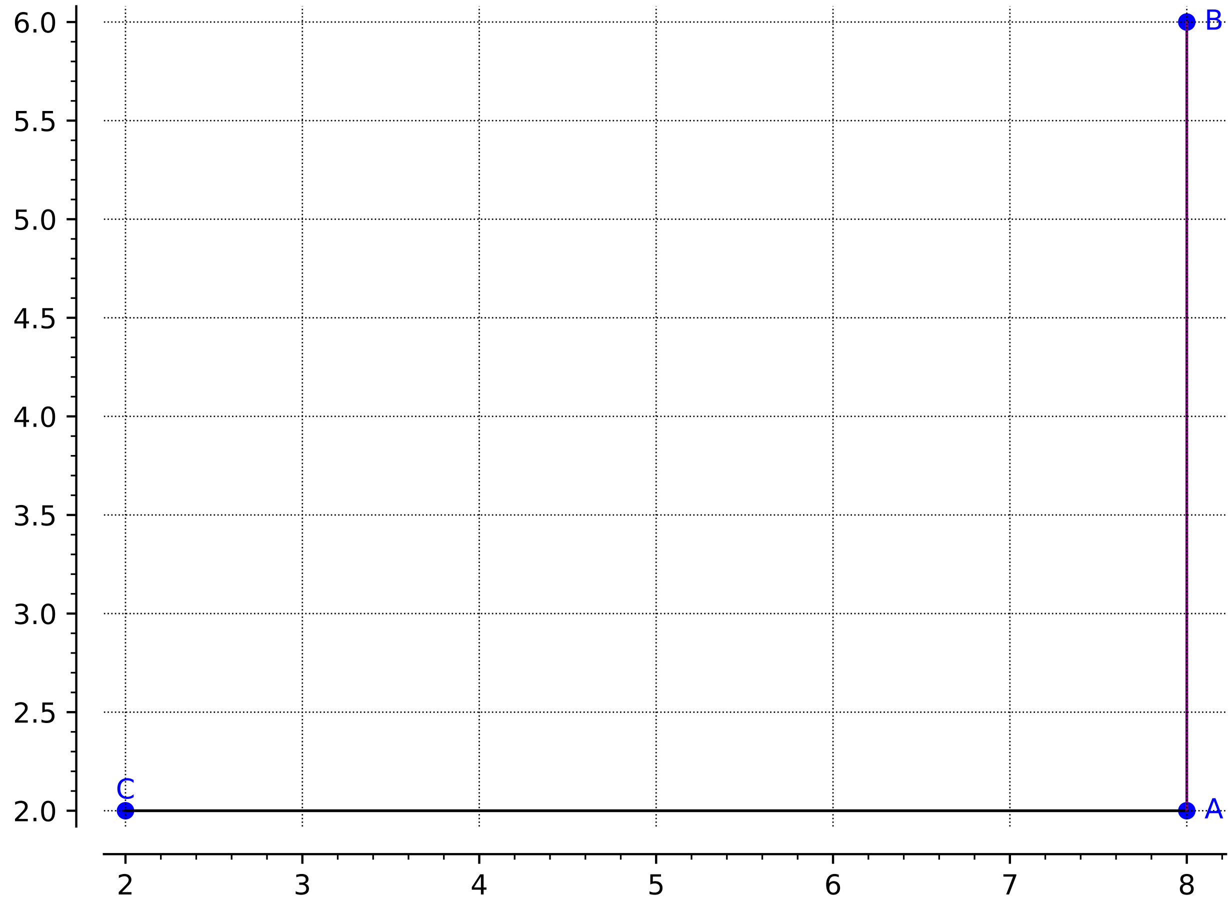


Figure 1.3.10

(a) What is the midpoint of the line segment AB?

A. $(16, 4)$
C. $(8, 8)$
B. $(8, 4)$
D. $(10, 2)$

(b) What is the midpoint of the line segment AC?

A. $(6, 0)$
C. $(6, 4)$
B. $(4, 4)$
D. $(5, 2)$

(c) Suppose we connect the two endpoints of the two line segments together, to create the new line segment, BC. Can you make an educated guess to where the midpoint of BC is?

A. $(10, 8)$

B. $(6, 4)$

C. $(5, 4)$

D. $(5, 2)$

(d) How can you test your conjecture? Is there a mathematical way to find the midpoint of any line segment?

Definition 1.3.11 The midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2), can be found by taking the average of the x and y values. Mathematically, the **midpoint formula** states that the midpoint of a line segment can be found by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$\Diamond$

Activity 1.3.12 Apply Definition 1.3.11 to calculate the midpoint of the following line segments.

(a) What is the midpoint of the line segment with endpoints $(-4, 5)$ and $(-2, -3)$?

A. $(3, 1)$

B. $(-3, 1)$

C. $(1, 1)$

D. $(1, 4)$

(b) What is the midpoint of the line segment with endpoints $(2, 6)$ and $(-6, -8)$?

A. $(-3, -1)$

B. $(-2, 0)$

C. $(-2, -1)$

D. $(4, 7)$

(c) Suppose C is the midpoint of AB and is located at $(9, 8)$. The coordinates of A are $(10, 10)$. What are the coordinates of B?

A. $(9.5, 9)$

B. $(11, 12)$

C. $(18, 16)$

D. $(8, 6)$

Activity 1.3.13 On a map, your friend Sarah's house is located at $(-2, 5)$ and your other friend Austin's house is at $(6, -2)$.

(a) How long is the direct path from Sarah's house to Austin's house?

(b) Suppose your other friend, Micah, lives in the middle between Sarah and Austin. What is the location of Micah's house on the map?

1.4 Absolute Value Equations and Inequalities (EQ4)

Objectives

- Solve linear equations involving an absolute value. Solve linear inequalities involving absolute values and express the answers graphically and using interval notation.

Remark 1.4.1 An absolute value, written $|x|$, is the non-negative value of x. If x is a positive number, then $|x| = x$. If x is a negative number, then $|x| = -x$.

Activity 1.4.2 Let's consider how to solve an equation when an absolute value is involved.

(a) Which values are solutions to the absolute value equation $|x| = 2$?

A. $x = 2$

B. $x = 0$

C. $x = -1$

D. $x = -2$

(b) Which values are solutions to the absolute value equation $|x - 7| = 2$?

A. $x = 9$

B. $x = 7$

C. $x = 5$

D. $x = -9$

(c) Which values are solutions to the absolute value equation $3|x-7|+5 = 11$? It may be helpful to rewrite the equation to isolate the absolute value.

A. $x = 7$

B. $x = -9$

C. $x = 5$

D. $x = 9$

Activity 1.4.3 Absolute value represents the distance a value is from 0 on the number line. So, $|x-7|=2$ means that the expression $x-7$ is 2 units away from 0.

(a) What values on the number line could $x-7$ equal?

A. $x=-7$

B. $x=-2$

C. $x=0$

D. $x=2$

E. $x=7$

(b) This gives us two separate equations to solve. What are those two equations?

A. $x-7=-7$

B. $x-7=-2$

C. $x-7=0$

D. $x-7=2$

E. $x-7=7$

(c) Solve each equation for x.

Remark 1.4.4 When solving an absolute value equation, begin by isolating the absolute value expression. Then rewrite the equation into two linear equations and solve. If $c > 0$,

$$|ax + b| = c$$

becomes the following two equations

$$ax + b = c \quad \text{and} \quad ax + b = -c$$

Activity 1.4.5 Solve the following absolute value equations.

(a) $|3x + 4| = 10$

A. $\{-2, 2\}$

B. $\left\{-\frac{14}{3}, 2\right\}$

C. $\{-10, 10\}$

D. No solution

(b) $3|x - 7| + 5 = 11$

A. $\{-2, 2\}$

B. $\{-9, 9\}$

C. $\{5, 9\}$

D. No solution

(c) $2|x + 1| + 8 = 4$

A. $\{-4, 4\}$

B. $\{-6, 6\}$

C. $\{5, 7\}$

D. No solution

Remark 1.4.6 Since the absolute value represents a distance, it is always a positive number. Whenever you encounter an isolated absolute value equation equal to a negative value, there will be no solution.

Activity 1.4.7 Just as with linear equations and inequalities, we can consider absolute value inequalities from equations.

(a) Which values are solutions to the absolute value inequality $|x - 7| \leq 2$?

A. $x = 9$

B. $x = 7$

C. $x = 5$

D. $x = -9$

(b) Rewrite the absolute value inequality $|x - 7| \leq 2$ as a compound inequality.

A. $0 \leq x - 7 \leq 2$

B. $-2 \leq x - 7 \leq 2$

C. $-2 \leq x - 7 \leq 0$

D. $2 \leq x \leq 7$

(c) Solve the compound inequality that is equivalent to $|x - 7| \leq 2$ found in part (b). Write the solution in interval notation.

A. $[7, 9]$

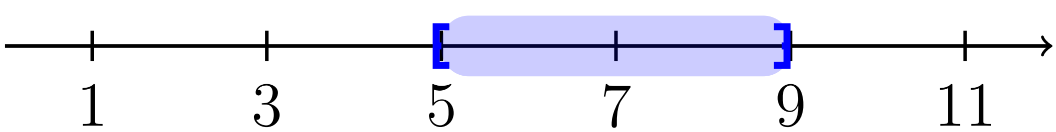
B. $[5, 9]$

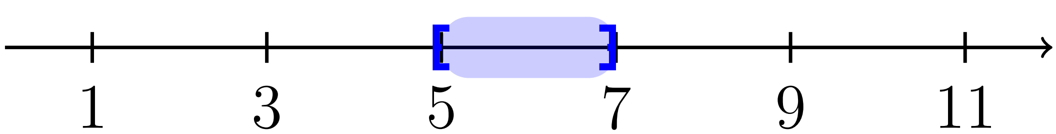
C. $[5, 7]$

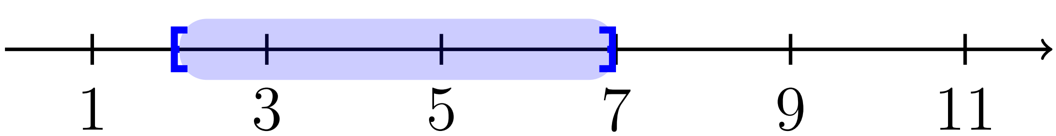
D. $[2, 7]$

(d) Draw the solution to $|x - 7| \leq 2$ on the number line.

A.


B.


C.


D.


Activity 1.4.8 Now let's consider another type of absolute value inequality.

(a) Which values are solutions to the absolute value inequality $|x-7| \geq 2$?

A. $x = 9$

B. $x = 7$

C. $x = 5$

D. $x = -9$

(b) Which two of the following inequalities are equivalent to $|x-7| \geq 2$.

A. $x - 7 \leq 2$

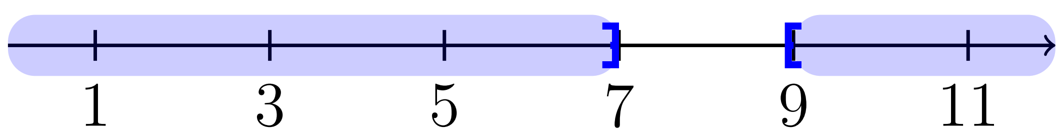
B. $x - 7 \leq -2$

C. $x - 7 \geq 2$

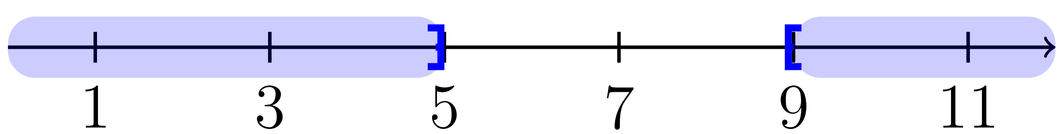
D. $x - 7 \geq -2$

(c) Solve the two inequalities found in part (b). Write the solution in interval notation and graph on the number line.

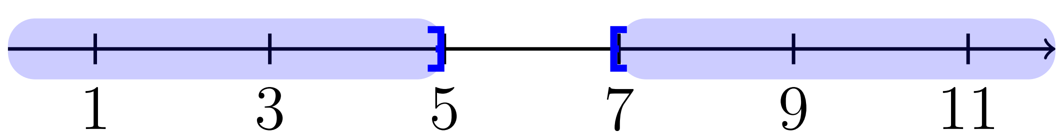
A. $(-\infty, 7] \cup [9, \infty)$



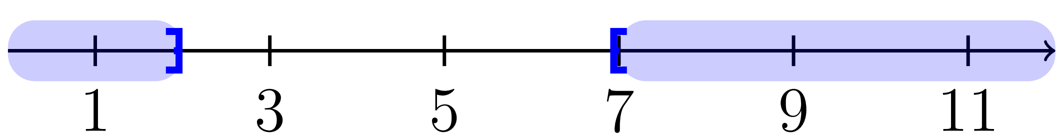
B. $(-\infty, 5] \cup [9, \infty)$



C. $(-\infty, 5] \cup [7, \infty)$



D. $(-\infty, 2] \cup [7, \infty)$



Definition 1.4.9 When solving an absolute value inequality, rewrite it as compound inequalities. Assume k is positive. $|x| < k$ becomes $-k < x < k$. $|x| > k$ becomes $x > k$ and $x < -k$. $\Diamond$

Activity 1.4.10 Solve the following absolute value inequalities. Write your solution in interval notation and graph on a number line.

(a) $|3x + 4| < 10$

(b) $3|x - 7| + 5 > 11$

1.5 Quadratic Equations (EQ5)

Objectives

- Solve quadratic equations using factoring, the square root property, completing the square, and the quadratic formula and express these answers in exact form.

Definition 1.5.1 A **quadratic equation** is of the form:

$$ax^2 + bx + c = 0$$

where a and b are coefficients (and $a \neq 0$), x is the variable, and c is the constant term. $\Diamond$

Quadratic Equations (EQ5)

Activity 1.5.2 Before beginning to solve quadratic equations, we need to be able to identify all various forms of quadratics.Which of the following is a quadratic equation?

A. $6 - x^2 = 3x$

B. $(2x - 1)(x + 3) = 0$

C. $4(x - 3) + 7 = 0$

D. $(x - 4)^2 + 1 = 0$

E. $5x^2 - 3x = 17 - 4x$

A, B, D, and E

Definition 1.5.3 To solve a quadratic equation, we will need to apply the **zero product property**, which states that if $a \cdot b = 0$, then either $a = 0$ or $b = 0$. In other words, you can only have a product of 0 if one (or both!) of the factors is 0. $\Diamond$

Activity 1.5.4 In this activity, we will look at how to apply the zero product property when solving quadratic equations.

(a) Which of the following equations can you apply Definition 1.5.3 as your first step in solving?

A. $2x^2 - 3x + 1 = 0$

B. $(2x + 1)(x + 1) = 0$

C. $3x^2 - 4 = 6x$

D. $x(3x + 5) = 0$

(b) Suppose you are given the quadratic equation, $(2x - 1)(x - 1) = 0$. Applying Definition 1.5.3 would give you:

A. $2x^2 - 3x + 1 = 0$

B. $(2x + 1) = 0$ and $(x + 1) = 0$

C. $(2x - 1) = 0$ and $(x - 1) = 0$

(c) After applying the zero product property, what are the solutions to the quadratic equation $(2x - 1)(x - 1) = 0$?

A. $x = -\frac{1}{2}$ and $x = 1$

B. $x = \frac{1}{2}$ and $x = -1$

C. $x = -\frac{1}{2}$ and $x = -1$

D. $x = \frac{1}{2}$ and $x = 1$

Remark 1.5.5 Notice in Activity 1.5.2 and Activity 1.5.4, that not all equations are set up ”nicely.” You will need to do some manipulation to get everything on one side (AND in factored form!) and 0 on the other ∗before∗ applying the zero product property.

Activity 1.5.6 Suppose you want to solve the equation $2x^2 + 5x - 12 = 0$, which is NOT in factored form.

(a) Which of the following is the correct factored form of $2x^2 + 5x - 12 = 0$?

A. $(2x - 3)(x - 4) = 0$

B. $(2x + 3)(x - 4) = 0$

C. $(2x + 3)(x + 4) = 0$

D. $(2x - 3)(x + 4) = 0$

(b) After applying Definition 1.5.3, which of the following will be a solution to $2x^2 + 5x - 12 = 0$?

A. $x = -\frac{3}{2}$ and $x = -4$

B. $x = \frac{3}{2}$ and $x = 4$

C. $x = -\frac{3}{2}$ and $x = 4$

D. $x = \frac{3}{2}$ and $x = -4$

Activity 1.5.7 Solve each of the following quadratic equations:

(a) $(2x-5)(x+7)=0$

(b) $3x(4x-1)=0$

(c) $3x^2-14x-5=0$

(d) $6-x^2=5x$

Activity 1.5.8 Suppose you are given the equation, $x^2 = 9$:

(a) How many solutions does this equation have?

A. 0

B. 1

C. 2

D. 3

(b) What are the solutions to this equation?

A. $x = 0$

B. $x = 3$

C. $x = 9, -9$

D. $x = 3, -3$

(c) How is this quadratic equation different than the equations we've solved thus far?

Definition 1.5.9 The **square root property** states that a quadratic equation of the form

$$x^2 = k^2$$

(where k is a nonzero number) will give solutions $x = k$ and $x = -k$. In other words, if we have an equation with a perfect square on one side and a number on the other side, we can take the square root of both sides to solve the equation. $\diamondsuit$

Activity 1.5.10 Suppose you are given the equation, $3x^2 - 8 = 4$:

(a) What would be the first step in solving $3x^2 - 8 = 4$?

A. Divide by 3 on both sides
B. Subtract 4 on both sides
C. Add 8 on both sides
D. Multiply by 3 on both sides

(b) Isolate the x^2 term and apply Definition 1.5.9 to solve for x.

(c) What are the solution(s) to $3x^2 - 8 = 4$?

A. $x = 6, -6$
B. $x = 2, -2$
C. $x = 0$
D. $x = 2$

Activity 1.5.11 Solve the following quadratic equations by applying the square root property (Definition 1.5.9).

(a) $3x^2 + 1 = 28$

(b) $5x^2 + 7 = 47$

(c) $2x^2 = -144$

Hint. Recall that when you have a negative number under a square root, that gives an imaginary number ($\sqrt{-1} = i$).

(d) $(x + 2)^2 + 3 = 19$

Hint. Isolate the binomial $(x + 2)$ first.

(e) $3(x - 4)^2 = 15$

Remark 1.5.12 Not all quadratic equations can be factored or can be solved by using the square root property. In the next few activities, we will learn two additional methods in solving quadratics.

Definition 1.5.13 Another method for solving a quadratic equation is known as **completing the square**. With this method, we add or subtract terms to both sides of an equation until we have a perfect square trinomial on one side of the equal sign and a constant on the other side. We then apply the square root property. Note: A perfect square trinomial is a trinomial that can be factored into a binomial squared. For example, $x^2 + 4x + 4$ is a perfect square trinomial because it can be factored into $(x+2)(x+2)$ or $(x+2)^2$. $\Diamond$

Activity 1.5.14 Let's work through an example together to solve $x^2+6x = 4$. (Notice that the methods of factoring and the square root property do not work with this equation.)

(a) In order to apply Definition 1.5.13, we first need to have a perfect square trinomial on one side of the equal sign. Which of the following number(s) could we add to the left side of the equation to create a perfect square trinomial?

A. 4

B. 9

C. -9

D. 2

(b) Add your answer from part *a* to the right side of the equation as well (i.e. whatever you do to one side of an equation you must do to the other side too!) and then factor the perfect square trinomial on the left side. Which equation best represents the equation now?

A. $(x+3)^2 = -5$

B. $(x-3)^2 = 13$

C. $(x+3)^2 = 13$

D. $(x-3)^2 = -5$

(c) Apply the square root property (Definition 1.5.9) to both sides of the equation to determine the solution(s). Which of the following is the solution(s) of $x^2 + 6x = 4$?

A. $3 + \sqrt{13}$ and $3 - \sqrt{13}$

B. $-3 + \sqrt{13}$ and $-3 - \sqrt{13}$

C. $3 + \sqrt{5}$ and $3 - \sqrt{5}$

D. $-3 + \sqrt{5}$ and $-3 - \sqrt{5}$

Remark 1.5.15 To complete the square, the leading coefficient, a (i.e., the coefficient of the x^2 term), must equal 1. If it does not, then factor the entire equation by a and then follow similar steps as in Activity 1.5.14.

Activity 1.5.16 Let's solve the equation $2x^2+8x-6=0$ by completing the square.

(a) Rewrite the equation so that all the terms with the variable x is on one side of the equation and a constant is on the other.

(b) Notice that the coefficient of the x^2 term is not 1. What could we factor the left side of the equation by so that the coefficient of the x^2 is 1?

(c) Once you factor the left side, what equation represents the equation you now have?

A. $2(x^2-8x)=-6$

B. $2(x^2-4x)=-6$

C. $2(x^2+4x)=6$

D. $2(x^2+8x)=6$

(d) Just like in Activity 1.5.14, let's now try and create the perfect square trinomial (inside the parentheses) on the left side of the equation. Which of the following number(s) could we add to the left side of the equation to create a perfect square trinomial?

A. 4

B. 8

C. -8

D. 2

(e) What would we need to add to the right-hand side of the equation to keep the equation balanced?

A. 4

B. 8

C. -8

D. 2

(f) Which of the following equation represents the quadratic equation you have now?

A. $2(x+2)^2=9$

B. $2(x-2)^2=2$

C. $2(x+2)^2=14$

D. $2(x-2)^2=14$

(g) Apply the square root property and solve the quadratic equation.

Activity 1.5.17 Solve the following quadratic equations by completing the square.

(a) $x^2 - 12x = -11$

(b) $x^2 + 2x - 33 = 0$

(c) $5x^2 + 29x = 6$

Definition 1.5.18 The last method for solving quadratic equations is the **quadratic formula** - a formula that will solve all quadratic equations! A quadratic equation of the form $ax^2+bx+c=0$ can be solved by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are real numbers and $a \neq 0$. $\diamondsuit$

Activity 1.5.19 Use the quadratic formula (Definition 1.5.18) to solve $x^2 + 4x = -3$.

(a) When written in standard form, what are the values of a, b, and c?

(b) When applying the quadratic formula, what would you get when you substitute a, b, and c?

A. $x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$

B. $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$

C. $x = \dfrac{-4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$

D. $x = \dfrac{4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$

(c) What is the solution(s) to $x^2 + 4x = -3$?

A. $x = -1, 3$

B. $x = 1, 3$

C. $x = -1, -3$

D. $x = 1, -3$

Activity 1.5.20 Use the quadratic formula (Definition 1.5.18) to solve $2x^2 - 13 = 7x$.

(a) When written in standard form, what are the values of a, b, and c?

(b) When applying the quadratic formula, what would you get when you substitute a, b, and c?

A. $x = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(13)}}{2(1)}$

B. $x = \dfrac{7 \pm \sqrt{(-7)^2 - 4(2)(-13)}}{2(2)}$

C. $x = \dfrac{-7 \pm \sqrt{(-7)^2 - 4(1)(-13)}}{2(1)}$

D. $x = \dfrac{7 \pm \sqrt{7^2 - 4(2)(-13)}}{2(2)}$

(c) What is the solution(s) to $2x^2 - 13 = 7x$?

A. $x = \frac{7+\sqrt{73}}{4}$ and $\frac{7-\sqrt{73}}{4}$

B. $x = \frac{7+\sqrt{153}}{4}$ and $\frac{-7-\sqrt{153}}{4}$

C. $x = \frac{-7+\sqrt{55}}{4}$ and $\frac{7-\sqrt{55}}{4}$

D. $x = \frac{-7+\sqrt{155}}{4}$ and $\frac{-7-\sqrt{155}}{4}$

Activity 1.5.21 Use the quadratic formula (Definition 1.5.18) to solve $x^2 = 6x - 12$.

(a) When written in standard form, what are the values of a, b, and c?

(b) When applying the quadratic formula, what would you get when you substitute a, b, and c?

A. $x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(12)}}{2(1)}$

B. $x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)}$

C. $x = \dfrac{-6 \pm \sqrt{(-6)^2 - 4(1)(-12)}}{2(1)}$

D. $x = \dfrac{6 \pm \sqrt{6^2 - 4(1)(-12)}}{2(1)}$

(c) Notice that the number under the square root is a negative. Recall that when you have a negative number under a square root, that gives an imaginary number ($\sqrt{-1} = i$). What is the solution(s) to $x^2 = 6x - 12$?

A. $x = 3 + i\sqrt{3}$ and $3 - i\sqrt{3}$

B. $x = 6 + i\sqrt{12}$ and $6 - i\sqrt{12}$

C. $x = -3 + i\sqrt{3}$ and $-3 - i\sqrt{3}$

D. $x = -6 + i\sqrt{12}$ and $-6 - i\sqrt{12}$

Activity 1.5.22 Solve the following quadratic equations by applying the quadratic formula (Definition 1.5.18).

(a) $2x^2 - 3x = 5$

(b) $4x^2 - 1 = -8x$

(c) $2x^2 - 7x - 13 = -10$

(d) $x^2 - 6x + 12 = 0$

Activity 1.5.23 Now that you have seen all the different ways to solve a quadratic equation, you will need to know WHEN to use which method. Are some methods better than others?

(a) Which is the best method to use to solve $5x^2 = 80$?

A. Factoring and Zero Product Property
B. Square Root Property
C. Completing the Square
D. Quadratic Formula

(b) Which is the best method to use to solve $5x^2 + 9x = -4$?

A. Factoring and Zero Product Property
B. Square Root Property
C. Completing the Square
D. Quadratic Formula

(c) Which is the best method to use to solve $3x^2 + 9x = 0$?

A. Factoring and Zero Product Property
B. Square Root Property
C. Completing the Square
D. Quadratic Formula

(d) Go back to parts a, b, and c and solve each of the quadratic equations. Would you still use the same method?

1.6 Rational Equations (EQ6)

Objectives

- Solve rational equations.

Rational Equations (EQ6)

Definition 1.6.1 An algebraic expression is called a **rational expression** if it can be written as the ratio of two polynomials, p and q.

An equation is called a **rational equation** if it consists of only rational expressions and constants. $\Diamond$

Observation 1.6.2 Technically, linear and quadratic equations are also rational equations. They are a special case where the denominator of the rational expressions is 1. We will focus in this section on cases where the denominator is not a constant; that is, rational equations where there are variables in the denominator.

With variables in the denominator, there will often be values that cause the denominator to be zero. This is a problem because division by zero is undefined. Thus, we need to be sure to exclude any values that would make those denominators equal to zero.

Activity 1.6.3 Which value(s) should be excluded as possible solutions to the following rational equations? Select all that apply.

(a)

$$\frac{2}{x+5} = \frac{x-3}{x-8} - 7$$

A. -7

B. -5

C. 2

D. 3

E. 8

(b)

$$\frac{x^2-6x+8}{x^2-4x+3} = 0$$

A. 0

B. 1

C. 2

D. 3

E. 4

Activity 1.6.4 Consider the rational equation

$$5 = -\frac{6}{x-2}$$

(a) What value should be excluded as a possible solution?

A. 5
B. 6
C. -6
D. 2
E. -2

(b) To solve, we begin by clearing out the fraction involved. What can we multiply each term by that will clear the fraction?

A. $x-5$
B. $x-6$
C. $x+6$
D. $x-2$
E. $x+2$

(c) Multiply each term by the expression you chose and simplify. Which of the following linear equations does the rational equation simplify to?

A. $5(x-5)=-6$
B. $5(x-6)=-6$
C. $5(x+6)=-6$
D. $5(x-2)=-6$
E. $5(x+2)=-6$

(d) Solve the linear equation. Check your answer using the original rational equation.

Activity 1.6.5 Consider the rational equation

$$\frac{4}{x+1} = -\frac{2}{x+6}$$

(a) What values should be excluded as possible solutions?

A. 2 and 4

B. 1 and 6

C. -1 and -6

D. 1 and 4

E. 2 and 6

(b) To solve, we'll once again begin by clearing out the fraction involved. Which of the following should we multiply each term by to clear out all of the fractions?

A. $x - 2$ and $x - 4$

B. $x - 1$ and $x - 6$

C. $x + 1$ and $x + 6$

D. $x - 1$ and $x - 4$

E. $x - 2$ and $x - 6$

(c) Multiply each term by the expressions you chose and simplify. Which of the following linear equations does the rational equation simplify to?

A. $4(x + 1) = -2(x + 6)$

B. $4(x + 6) = -2(x + 1)$

C. $4(x + 1)(x + 6) = -2(x + 1)(x + 6)$

D. $4(x + 1) = -2(-x - 6)$

E. $4(x + 6) = -2(-x - 1)$

(d) Solve the linear equation. Check your answer using the original rational equation.

Observation 1.6.6 In Activity 1.6.5, you may have noticed that the resulting linear equation looked like the result of cross-multiplying. This is no coincidence! Cross-multiplying is a method of clearing out fractions that works specifically when the equation is in proportional form: $\frac{a}{b} = \frac{c}{d}$.

Activity 1.6.7 Consider the rational equation

$$\frac{x}{x+2} = -\frac{2}{x+2} - \frac{2}{5}$$

(a) What value(s) should be excluded as possible solutions?

(b) To solve, we'll once again begin by clearing out the fraction involved. Which of the following should we multiply each term by to clear out all of the fractions?

A. $x + 2$, $x + 2$, and 5

B. $x + 2$ and 5

C. $x + 2$

D. 5

(c) Multiply each term by the expressions you chose and simplify. You should end up with a linear equation.

(d) Solve the linear equation. Check your answer using the original rational equation.

Observation 1.6.8 Activity 1.6.7 demonstrates why it is so important to determine excluded values and check our answers when solving rational equations. Just because a number is a solution to the *linear* equation we found, it doesn't mean it is automatically a solution to the *rational* equation we started with.

Activity 1.6.9 Consider the rational equation

$$\frac{2x}{x-1} - \frac{3}{x-3} = \frac{x^2-11x+18}{x^2-4x+3}$$

(a) What values should be excluded as possible solutions? Select all that apply.

A. 0

B. 1

C. 2

D. 3

E. 9

(b) To solve, we'll begin by clearing out any fractions involved. Which of the following should we multiply each term by to clear out all of the fractions?

A. $x-1$

B. $x-1$ and $x-3$

C. $x-1$, $x-3$, and x^2-4x+3

D. $x-1$ and x^2-4x+3

E. $x-3$ and x^2-4x+3

(c) Multiply each term by the expressions you chose and simplify. Notice that the result is a quadratic equation. Which of the following quadratic equations does the rational equation simplify to?

A. $x^2+2x-15=0$

B. $x^2-11x+18=0$

C. $x^2-9x-9=0$

D. $x^2-13x+21=0$

(d) Solve the quadratic equation. Check your answer using the original rational equation. What are the solutions to the rational equation?

A. $x=3$ and $x=-5$

B. $x=-3$ and $x=5$

C. $x=3$

D. $x = -5$

E. $x = -3$

F. $x = 5$

Activity 1.6.10 Consider the rational equation

$$\frac{2x}{x-2} - \frac{x^2+21x-15}{x^2+3x-10} = \frac{-6}{x+5}$$

(a) What values should be excluded as possible solutions?

(b) What expression(s) should we multiply by to clear out all of the fractions?

(c) Multiply each term by the expressions you chose and simplify. Your result should be a quadratic equation.

(d) Solve the quadratic equation. Check your answer using the original rational equation. What are the solutions to the rational equation?

Activity 1.6.11 Solve the following rational equations.

(a) $\dfrac{4}{x} + 9 = 16$

(b) $-5 = \dfrac{2}{x-4}$

(c) $\dfrac{-3}{x-10} = \dfrac{x}{x-6}$

(d) $\dfrac{x+2}{x-3} + \dfrac{x}{2x-1} = 6$

1.7 Quadratic and Rational Inequalities (EQ7)

Objectives

- Solve quadratic inequalities and express the solution graphically and with interval notation. Solve rational inequalities and express the solution graphically and using interval notation.

Remark 1.7.1 In Section 1.5 and Section 1.6 we learned how to solve quadratic and rational equations. In this section, we use these skills to solve quadratic and rational *inequalities*.

Activity 1.7.2 Consider the quadratic inequality

$$x^2 - 4x - 32 > 0.$$

(a) Use a graphing utility to graph the function $f(x) = x^2 - 4x - 32$. Which part of the graph represents where $x^2 - 4x - 32 > 0$?

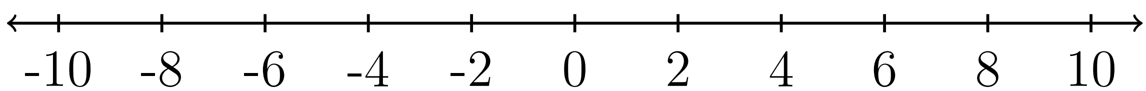
(b) Which pieces of information about $f(x) = x^2 - 4x - 32$ were needed to answer part (a)?

A. The y-intercept

B. The x-intercepts

C. The minimum value

(c) Use algebra to find the x-intercepts of $f(x) = x^2 - 4x - 32$ and mark them on a number line. Then, shade the part of the number line where $x^2 - 4x - 32 > 0$.



(d) Now use interval notation to express where $x^2 - 4x - 32 > 0$.

A. $(-4, 8)$

B. $[-4, 8]$

C. $(-\infty, -4) \cup (8, \infty)$

D. $(-\infty, -4] \cup [8, \infty)$

Definition 1.7.3 A **sign chart** is a number line representing the x-axis that shows where a function is positive or negative. Instead of shading, which can be ambiguous, it is often decorated with a '+' or a '-' to indicate which regions are positive or negative. For example, a sign chart for $f(x) = x^2 - 4x - 32$ is below.

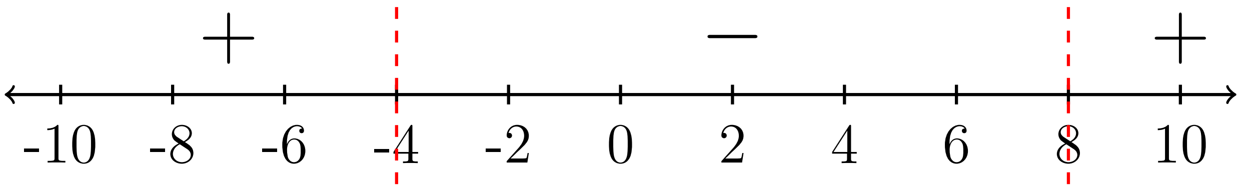


Figure 1.7.4 A sign chart for the function $f(x) = x^2 - 4x - 32$.

$\Diamond$

Activity 1.7.5 Solve the quadratic inequality algebraically

$$2x^2 - 28 < 10x.$$

Write your solution using interval notation.

A. $(-\infty, -2) \cup (7, \infty)$

B. $(-\infty, -7) \cup (2, \infty)$

C. $(-7, 2)$

D. $(-2, 7)$

Activity 1.7.6 Solve the inequality

$$-2x^2 - 10x - 10 \geq 6x + 20.$$

Write your solution using interval notation.

A. $[-5, -3]$

B. $(-\infty, -5] \cup [-3, \infty)$

C. $[3, 5]$

D. $(-\infty, 3] \cup [5, \infty)$

Activity 1.7.7 Consider the rational inequality

$$\frac{4x+3}{x+2} > x.$$

(a) For which of the following functions will a graph help us solve the rational inequality above?

A. $f(x) = \dfrac{4x+3}{x+2}$

B. $g(x) = \dfrac{4x+3}{x+2} - x$

C. $h(x) = x - \dfrac{4x+3}{x+2}$

(b) Use a graphing utility to graph the function $g(x) = \frac{4x+3}{x+2} - x$. Which part of the graph represents where $\frac{4x+3}{x+2} - x > 0$?

(c) Simplify $\frac{4x+3}{x+2} - x$ into a single rational expression.

A. $\dfrac{4x+3}{x+2}$

B. $\dfrac{3x+3}{x+2}$

C. $\dfrac{x^2+6x+3}{x+2}$

D. $\dfrac{-x^2+2x+3}{x+2}$

(d) How do these values you found in part (b) relate to the numerator and the denominator of the combined rational function in part (c)?

Hint. Factor the numerator.

(e) For what values is the original inequality a true statement?

A. $x < -2$ and $-1 < x < 3$

B. $-2 < x < -1$ and $x > 3$

C. $-2 < x < -1$

D. $1 < x < 3$

(f) How can we express the answers to part (e) for the rational inequality using interval notation?

A. $(-\infty, -2) \cup (-1, 3)$

B. $(-2, -1) \cup (3, \infty)$

C. $(-2, -1)$

D. $(1, 3)$

Quadratic and Rational Inequalities (EQ7)

Definition 1.7.8 The values on the x-axis where a function is equal to zero or undefined are called **partition values.** $\Diamond$

Activity 1.7.9 Solve the rational inequality

$$\frac{x+8}{x-2} \leq \frac{x+10}{x+5}.$$

(a) Write the solution using interval notation.

A. $(-\infty, -12) \cup [-5, 2]$

B. $(-\infty, -12] \cup (-5, 2)$

C. $(-12, -5] \cup [2, \infty)$

D. $[-12, -5) \cup (2, \infty)$

(b) Compare the interval notation from Activity 1.7.7 to the interval notation for this activity. When do we include the partition values in the answer with a bracket?

Chapter 2

Functions (FN)

Objectives

How do we express relationships between two quantities?
By the end of this chapter, you should be able to...

1. Determine if a relation, equation, or graph defines a function using the definition as well as the vertical line test.

2. Use and interpret function notation to evaluate a function for a given input value and find a corresponding input value given an output value.

3. Use the graph of a function to find the domain and range in interval notation, the x- and y-intercepts, the maxima and minima, and where it is increasing and decreasing using interval notation.

4. Apply transformations including horizontal and vertical shifts, stretches, and reflections to a function. Express the result of these transformations graphically and algebraically.

5. Find the sum, difference, product, quotient, and composition of two or more functions and evaluate them.

6. Find the inverse of a one-to-one function.

2.1 Introduction to Functions (FN1)

Objectives

- Determine if a relation, equation, or graph defines a function using the definition as well as the vertical line test.

Definition 2.1.1 A **relation** is a relationship between sets of values. Relations in mathematics are usually represented as ordered pairs: (input, output) or (x, y). When observing relations, we often refer to the x-values as the **domain** and the y-values as the **range**. $\Diamond$

Definition 2.1.2 Mapping Notation (also known as an arrow diagram) is a way to show relationships visually between sets. For example, suppose you are given the following ordered pairs: $(3, -8), (4, 6)$, and $(2, -1)$. Each of the x-values "map onto" a y-value and can be visualized in the following way:

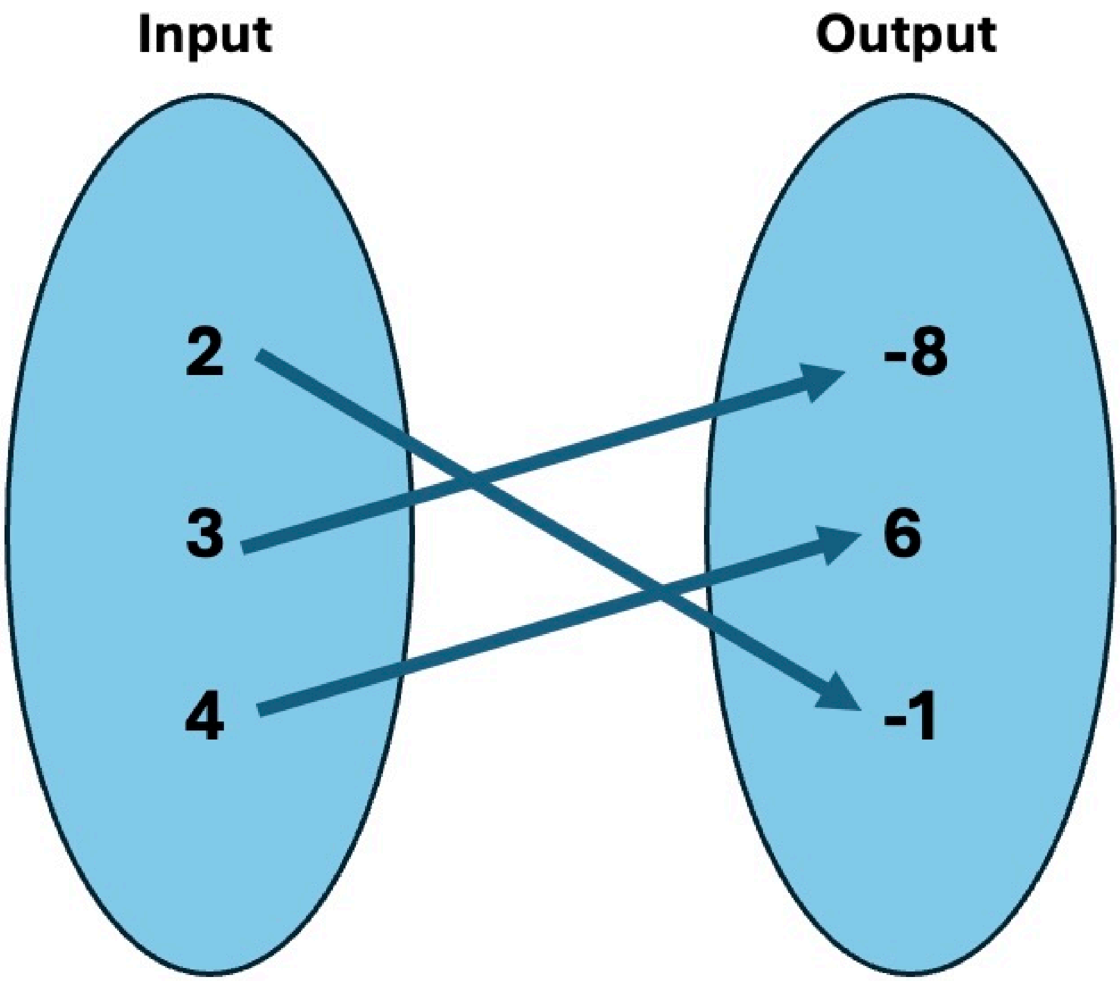


Figure 2.1.3 Every x-value from the ordered pair list is listed in the input set and every y-value is listed in the output set. An arrow is drawn from every x-value to its corresponding y-value.

Notice that an arrow is used to indicate which x-value is mapped onto its corresponding y-value. ◊

Activity 2.1.4 Use mapping notation to create a visual representation of the following relation.

$$(-1, 5), (2, 6), (4, -2)$$

(a) What is the domain?

A. $\{5, 6, -2\}$

B. $\{-1, 2, 4\}$

C. $\{-2, -1, 2, 4, 5, 6\}$

(b) What is the range?

A. $\{5, 6, -2\}$

B. $\{-1, 2, 4\}$

C. $\{-2, -1, 2, 4, 5, 6\}$

Activity 2.1.5 Use mapping notation to create a visual representation of the following relation.

$$(6,4),(3,4),(6,5)$$

(a) What is the domain?

A. $\{3,6\}$

B. $\{6,3,6\}$

C. $\{3,4,5,6\}$

D. $\{4,5\}$

(b) What is the range?

A. $\{3,6\}$

B. $\{6,3,6\}$

C. $\{3,4,5,6\}$

D. $\{4,5\}$

Activity 2.1.6 Use mapping notation to create a visual representation of the following relation.

$$(1,2),(-5,2),(-7,2)$$

(a) What is the domain?

A. $\{2,2,2\}$

B. $\{-7,-5,1,2\}$

C. $\{-7,-5,1\}$

D. $\{2\}$

(b) What is the range?

A. $\{2,2,2\}$

B. $\{-7,-5,1,2\}$

C. $\{-7,-5,1\}$

D. $\{2\}$

Remark 2.1.7 Notice that in Activity 2.1.4, Activity 2.1.5, and Activity 2.1.6, each set represents a very different relationship. Many concepts in mathematics will depend on particular relationships, so it is important to be able to visualize relationships and compare them.

Definition 2.1.8 A **function** is a relation where every input (or x-value) is mapped onto *exactly one* output (or y-value).

Note that all functions are relations but not all relations are functions!

$\Diamond$

Activity 2.1.9 Relations can be expressed in multiple ways (ordered pairs, tables, and verbal descriptions).

(a) Let's revisit some of the sets of ordered pairs we've previously explored in Activity 2.1.4, Activity 2.1.5, and Activity 2.1.6. Which of the following sets of ordered pairs represent a function?

A. $(-1, 5), (2, 6), (4, -2)$

B. $(6, 4), (3, 4), (6, 5)$

C. $(1, 2), (-5, 2), (-7, 2)$

D. $(-1, 2), (-1, 9), (1, 9)$

(b) Note that relations can be expressed in a table. A table of values is shown below. Is this an example of a function? Why or why not?

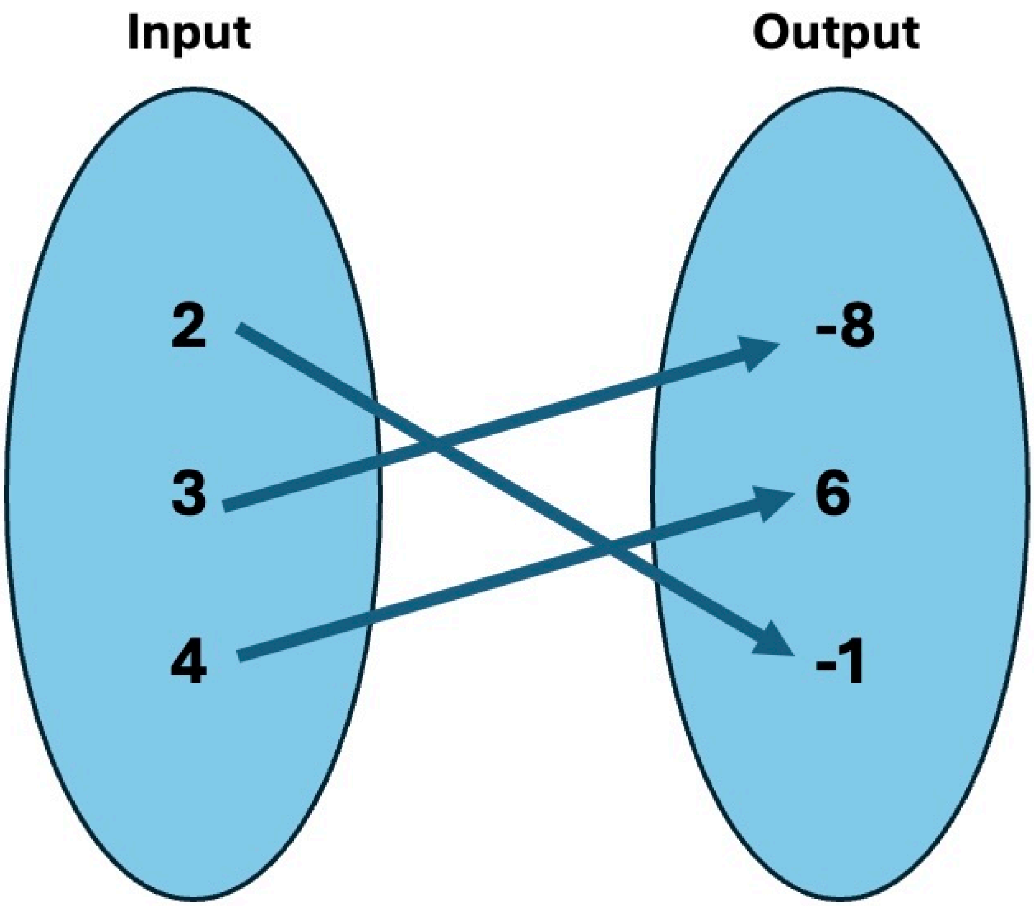
x	y
-5	-2
-4	-5
-2	8
8	-4
8	1

(c) Relations can also be expressed in words. Suppose you are looking at the amount of time you spend studying versus the grade you earn in your Algebra class. Is this an example of a function? Why or why not?

Remark 2.1.10 Notice that when trying to determine if a relation is a function, we often have to rely on looking at the domain and range values. Thus, it is important to be able to idenfity the domain and range of any relation!

Activity 2.1.11 For each of the given functions, determine the domain and range.

(a) $(-4, 3), (-1, 8), (7, 4), (1, 9)$



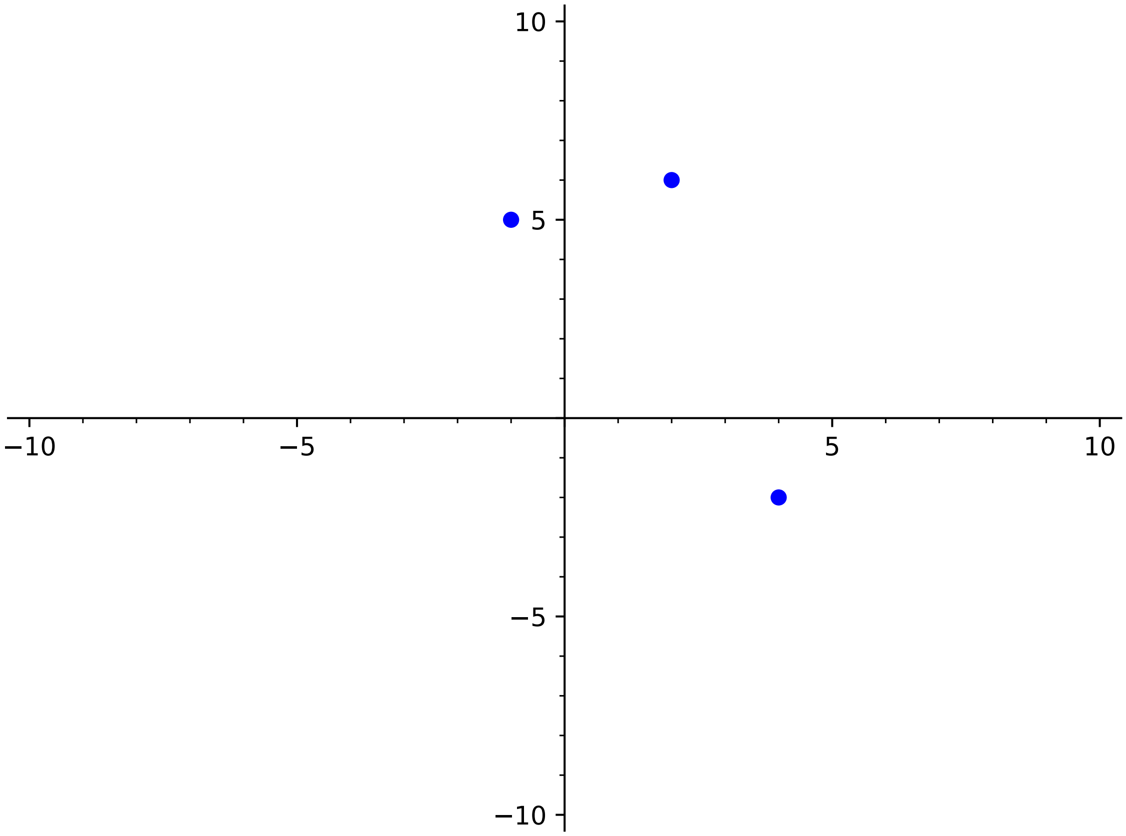
(b)

(c)

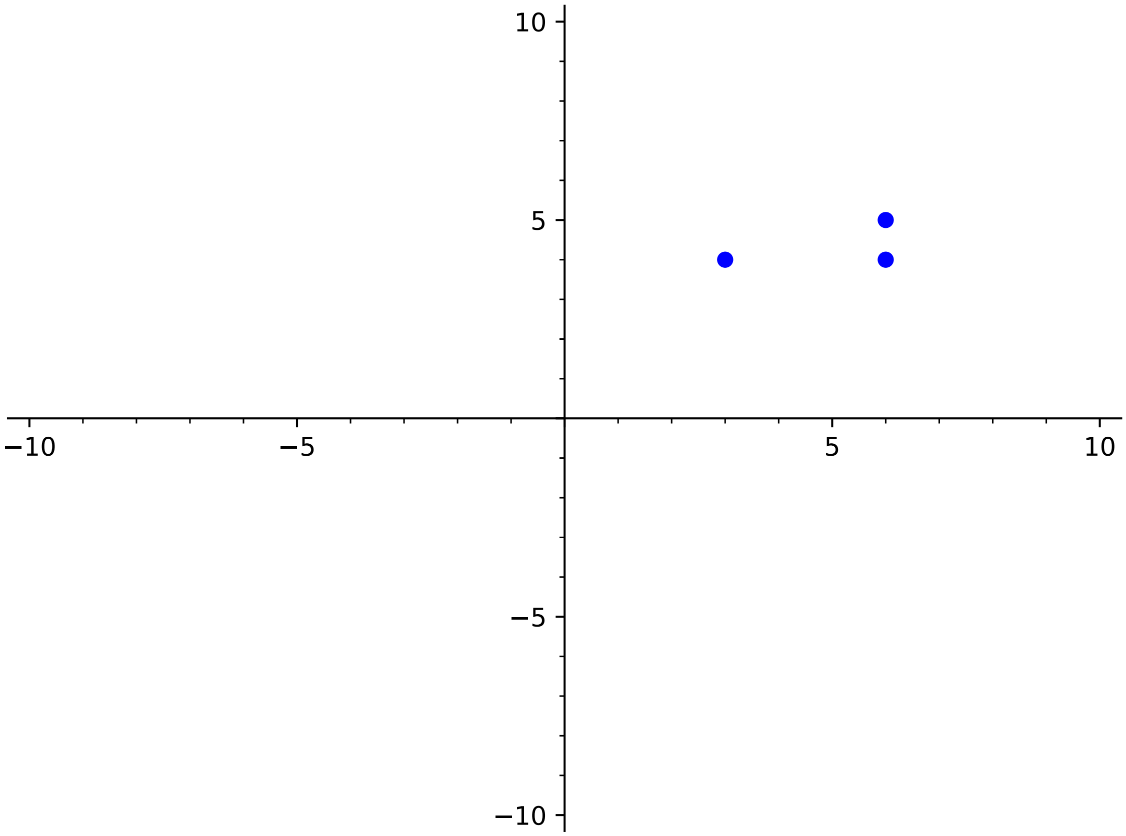
x	y
-2	5
0	4
3	6
8	1

(d) The amount of time you spend studying versus the grade you earn in your Algebra class.

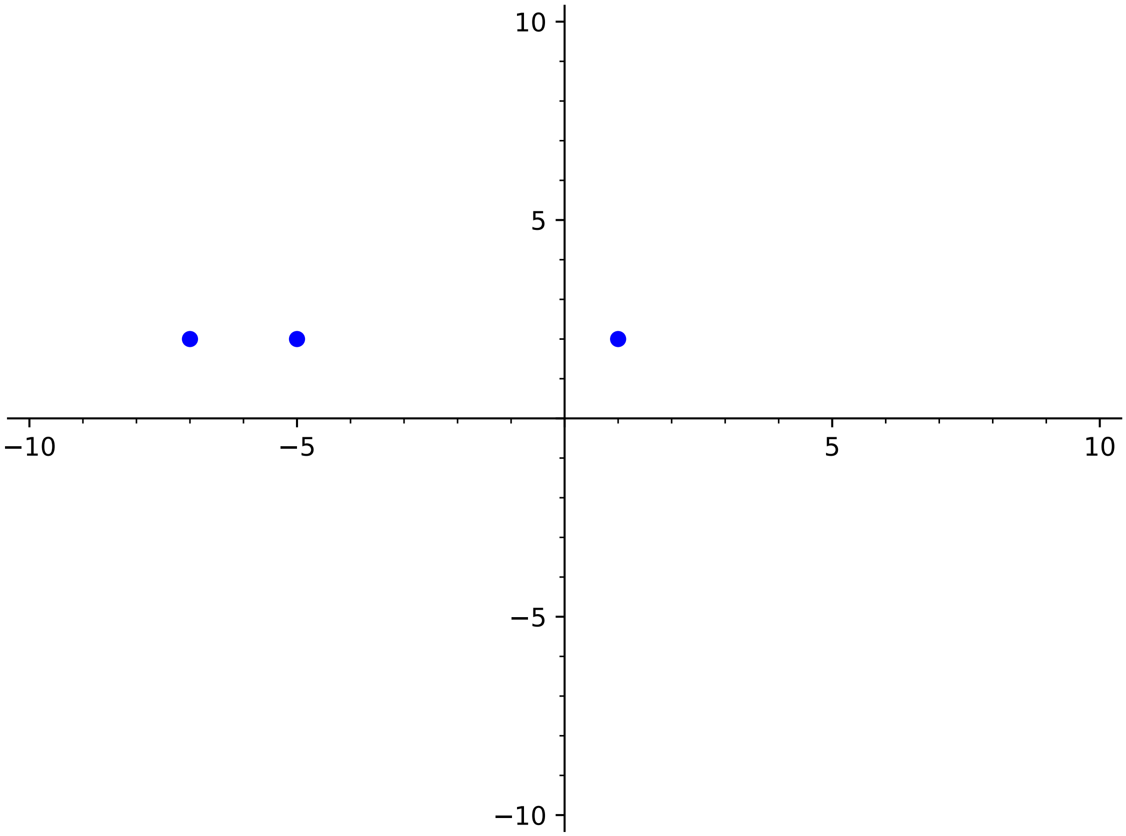
Activity 2.1.12 Determine whether each of the following relations is a function.



(a)



(b)



(c)

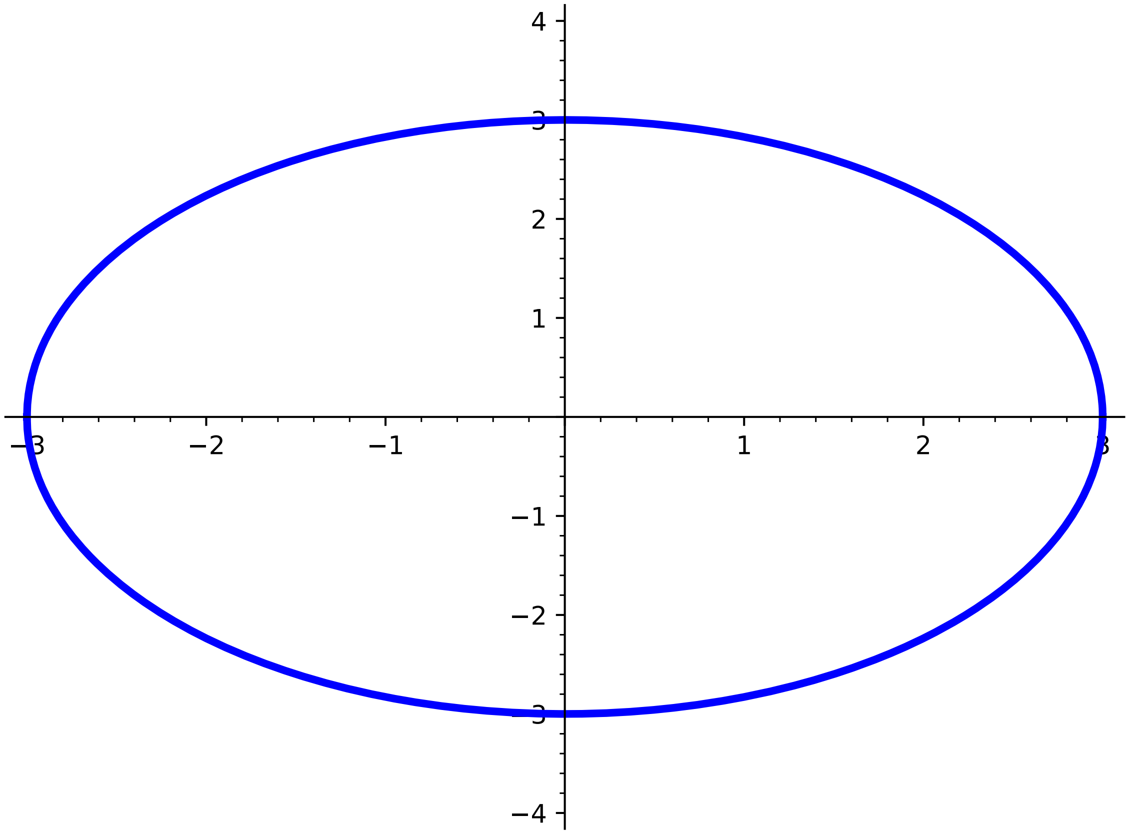
Remark 2.1.13 You probably noticed (in Activity 2.1.12) that when the graph has points that "line up" or are on top of each other, they have the same x-values. When this occurs, this shows that the same x-value has two different outputs (y-values) and that the relation is not a function.

Definition 2.1.14 The **vertical line test** is a method used to determine whether a relation on a graph is a function.

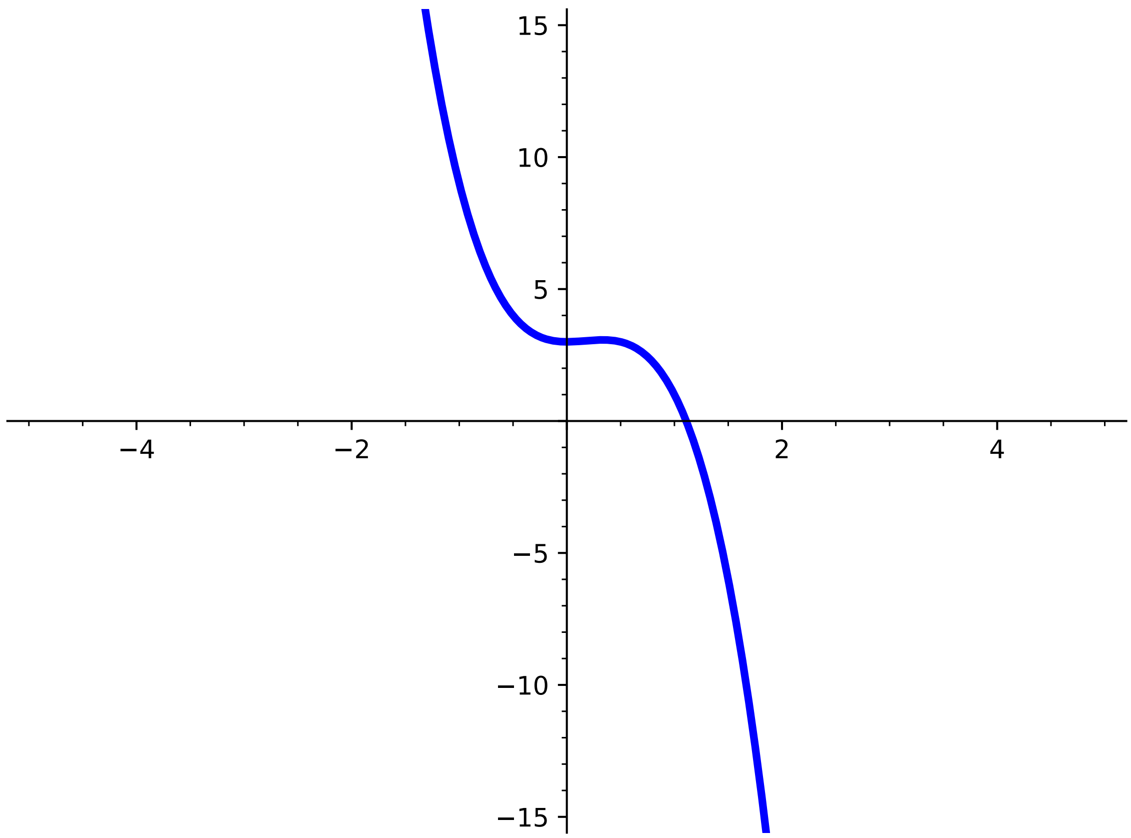
Start by drawing a vertical line anywhere on the graph and observe the number of times the relation on the graph intersects with the vertical line. If every possible vertical line intersects the graph at only one point, then the relation is a function. If, however, the graph of the relation intersects a vertical line more than once (anywhere on the graph), then the relation is not a function. $\Diamond$

Activity 2.1.15 Use the vertical line test (Definition 2.1.14) to determine whether each graph of a relation represents a function.

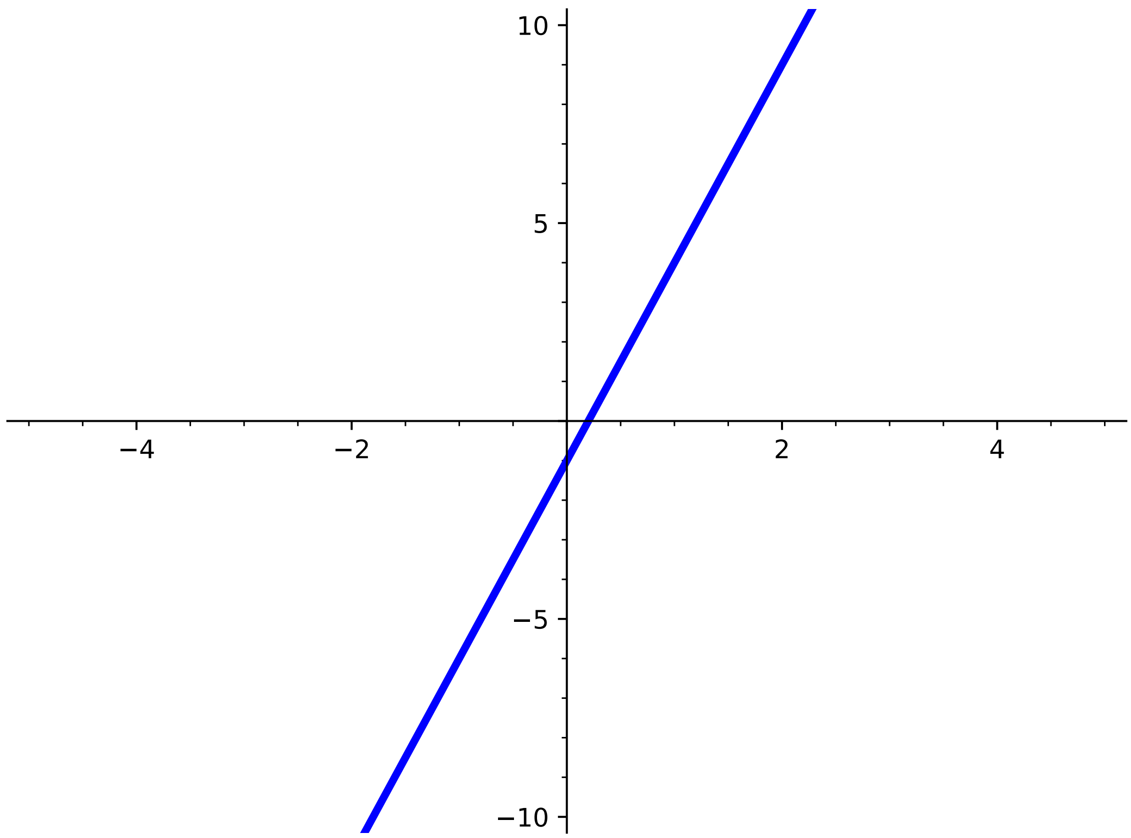
(a)



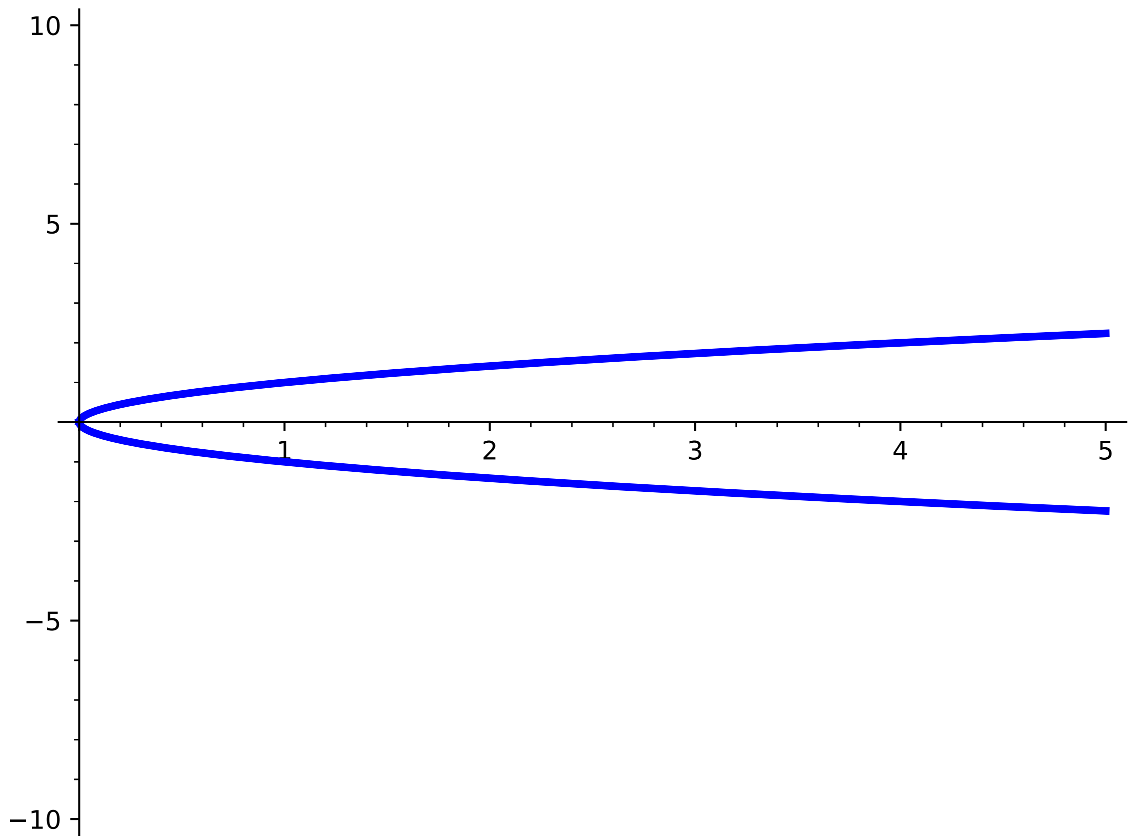
(b)



(c)



(d)



Activity 2.1.16 Let's explore how to determine whether an equation represents a function.

(a) Suppose you are given the equation $x = y^2$.

- If $x = 4$, what kind of y-values would you get for $x = y^2$?
- Based on this information, do you think $x = y^2$ is a function?

(b) Suppose you are given the equation $y = 3x^2 + 2$.

- If $x = 4$, what kind of y-values would you get for $y = 3x^2 + 2$?
- Based on this information, do you think $y = 3x^2 + 2$ is a function?

(c) Suppose you are given the equation $x^2 + y^2 = 25$.

- If $x = 4$, what kind of y-values would you get for $x^2 + y^2 = 25$?
- Based on this information, do you think $x^2 + y^2 = 25$ is a function?

(d) Suppose you are given the equation $y = -4x - 3$.

- If $x = 4$, what kind of y-values would you get for $y = -4x - 3$?
- Based on this information, do you think $y = -4x - 3$ is a function?

(e) How can you look at an equation to determine whether or not it is a function?

Remark 2.1.17 Notice that Activity 2.1.16 shows that equations with a y^2 term generally do not define functions. This is because to solve for a squared variable, you must consider both positive and negative inputs. For example, both $2^2 = 4$ and $(-2)^2 = 4$.

Activity 2.1.18 It's important to be able to determine the domain of any equation, especially when thinking about functions. Answer the following questions given the equation $y = \sqrt{x}$.

(a) What values of x would give an error (if any)?

A. -2

B. 0

C. 4

D. -5

(b) Based on this information, for what values of x would the equation exist?

A. -2

B. 0

C. 4

D. -5

(c) How can we represent the domain of this equation in interval notation?

A. $(-\infty, 0)$

B. $[0, \infty)$

C. $(0, 0)$

D. $(-\infty, \infty)$

Activity 2.1.19 Answer the following questions given the equation $y = -5x + 1$.

(a) What values of x would give an error (if any)?

A. -2

B. 0

C. 4

D. -5

(b) Based on this information, for what values of x would the equation exist?

A. -2

B. 0

C. 4

D. -5

(c) How can we represent the domain of this equation in interval notation?

A. $(-\infty, 0)$

B. $(0, \infty)$

C. $(-5, 1)$

D. $(-\infty, \infty)$

Activity 2.1.20 Answer the following questions given the equation $y = \frac{3}{x-5}$.

(a) What values of x would give an error (if any)?

A. -3

B. 0

C. -4

D. 5

(b) Based on this information, for what values of x would the equation exist?

A. -3

B. 0

C. -4

D. 5

(c) How can we represent the domain of this equation in interval notation?

A. $(-\infty, 5)$

B. $(5, \infty)$

C. $(-5, 5)$

D. $(-\infty, 5)\mathrm{U}(5, \infty)$

Remark 2.1.21 When determining the domain of an equation, it is often easier to first find values of x that make the function undefined. Once you have those values, then you know that x can be any value but those.

2.2 Function Notation (FN2)

Objectives

- Use and interpret function notation to evaluate a function for a given input value and find a corresponding input value given an output value.

Remark 2.2.1 As we saw in the last section, we can represent functions in many ways, like using a set of ordered pairs, a graph, a description, or an equation. When describing a function with an equation, we will often use function notation.

If y is written as a function of x, like in the equation

$$y = x + 5,$$

we can replace the y with $f(x)$ and get the function notation

$$f(x) = x + 5.$$

The x is the input variable, and $f(x)$ is the y-value or output that corresponds to x.

Generally, we use the letter f for functions. Other letters are okay as well; $g(x)$ and $h(x)$ are common. If we are using multiple functions at one time, we often denote them with different letters so we can refer to one without any confusion as to which function we mean.

Activity 2.2.2 Rewrite the following equations using function notation. In each case, assume y is a function of the variable x.

(a) $y = 2x + 14$

(b) $y + x = 3x^2 - 5$

(c) $\dfrac{2}{x} - x^4 = y - 5$

Activity 2.2.3 Let $f(x) = 3x^2 - 4x + 1$. Find the value of $f(x)$ for the given values of x.

Table 2.2.4

x	$f(x)$
-5	
$-\frac{1}{2}$	
0	
2	
10	

Remark 2.2.5 If we are asked to find the value of $f(x)$ for a certain x-value, say $x = 5$, we use the notation $f(5)$ to indicate that.

Activity 2.2.6 Let $f(x)$, $g(x)$, and $h(x)$ be defined as shown.

$$f(x) = 3x^2 - 4x + 1$$
$$g(x) = \sqrt{13 - x^2}$$
$$h(x) = \frac{x^2 - 6x + 8}{x^2 - 4x + 3}$$

Find the following, if they exist.

(a) $f(-4)$, $f(0)$, and $f(2)$

(b) $g(0)$, $g(2)$, and $g(8)$

(c) $h(3)$, $h(4)$, and $h(10)$

Remark 2.2.7 Sometimes functions are made up of multiple functions put together. We call these **piecewise functions**. Each piece is defined for only a certain interval, and these intervals do not overlap. When evaluating a piecewise function at a given x-value, we first need to find the interval that includes the x-value, and then plug in to the corresponding function piece.

Activity 2.2.8 Let $f(x)$ be a piecewise function as shown below.

$$f(x) = \begin{cases} x^2 + 3, & x < 5 \\ 9 - 2x, & x \geq 5 \end{cases}$$

(a) On which interval from the piecewise function does the value $x = 1$ belong?

A. $x < 5$ B. $x \leq 5$ C. $x > 5$ D. $x \geq 5$

(b) Find $f(1)$.

A. 3 B. 4 C. 5 D. 6 E. 7

(c) On which interval from the piecewise function does the value $x = 5$ belong?

A. $x < 5$ B. $x \leq 5$ C. $x > 5$ D. $x \geq 5$

(d) Find $f(5)$.

A. -10 B. -5 C. -1 D. 17 E. 28

Remark 2.2.9 We've been practicing evaluating functions at specific numeric values. It's also possible to evaluate a function given an expression involving variables.

Activity 2.2.10 Let $g(x) = x^2 - 3x$.

(a) Find $g(a)$.

A. $(ax)^2 - 3ax$

B. $a^2 - 3a$

C. $a(x^2 - 3x)$

D. $ax^2 - 3ax$

E. $a - 3$

(b) Find $g(x + h)$.

A. $x^2 - 3x + h$

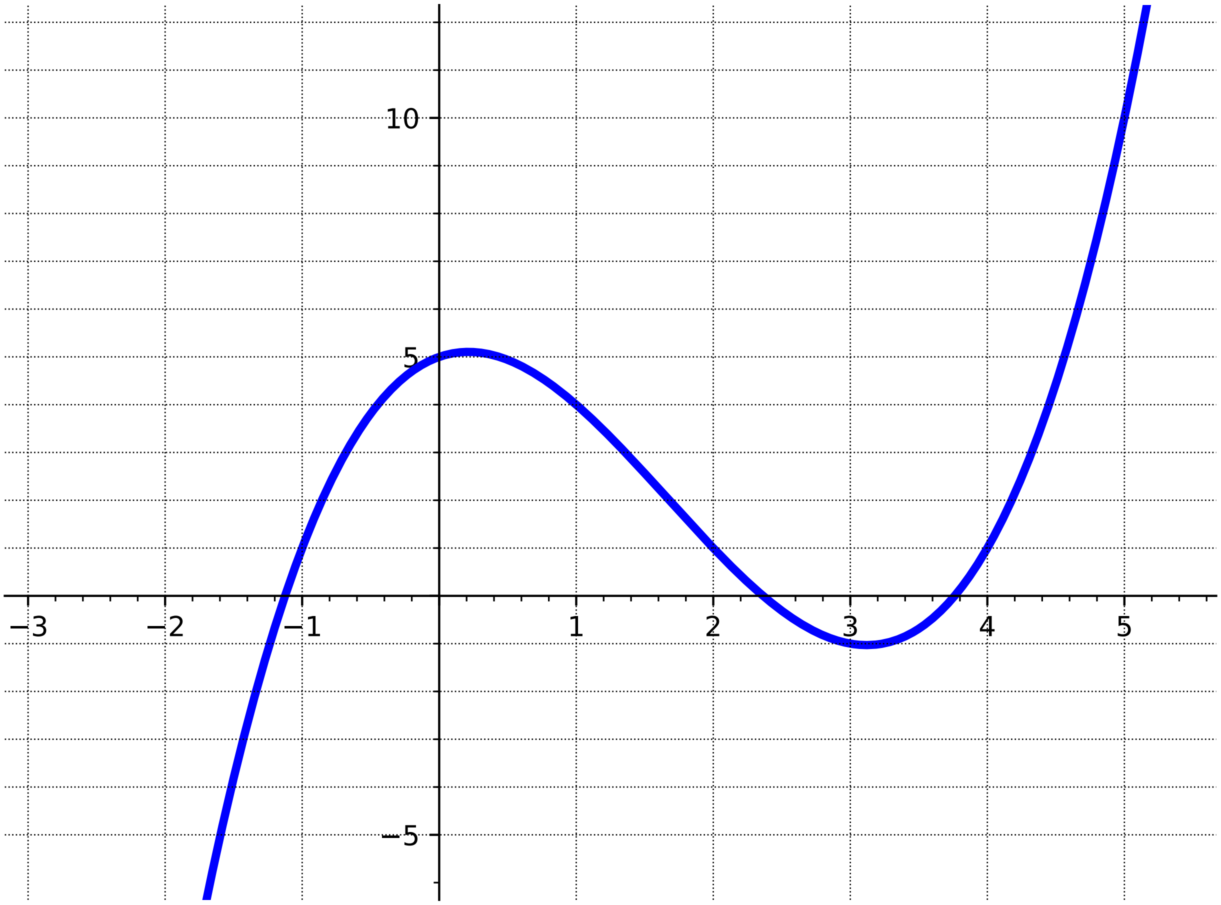
B. $(x + h)^2 - 3x$

C. $(x + h)^2 - 3(x + h)$

D. $x^2 - 3(x + h)$

Remark 2.2.11 We should also be able to look at a graph of a function and evaluate it for different values of x. The next activity explores that.

Activity 2.2.12 Let $f(x)$ be the function graphed below.



(a) Find $f(1)$.

A. -4 B. -2 C. 0 D. 2 E. 4

(b) Find $f(3)$.

A. -1 B. 0 C. 1 D. 2 E. 3 F. 4

(c) For which x-value(s) does $f(x) = 1$?

A. -1 B. 0 C. 1 D. 2 E. 3 F. 4

(d) For which x-value(s) does $f(x) = 4$. Estimate as needed!

Activity 2.2.13 In these activities, we are flipping the question around. This time we know what the function equals at some x-value, and we want to recover that x-value (or values!).

(a) Let $h(x) = 5x + 7$. Find the x-value(s) such that $h(x) = -13$.

(b) Let $f(x) = x^2 - 3x - 9$. Find the x-value(s) such that $f(x) = 9$.

Activity 2.2.14 Ellie has \$13 in her piggy bank, and she gets an additional \$1.50 each week for her allowance. Assuming she does not spend any money, the total amount of allowance, $A(w)$, she has after w weeks can be modeled by the function

$$A(w) = 13 + 1.50w.$$

(a) How much money will be in her piggy bank after 5 weeks?

(b) After how many weeks will she have \$40 in her piggy bank?

2.3 Characteristics of a Function's Graph (FN3)

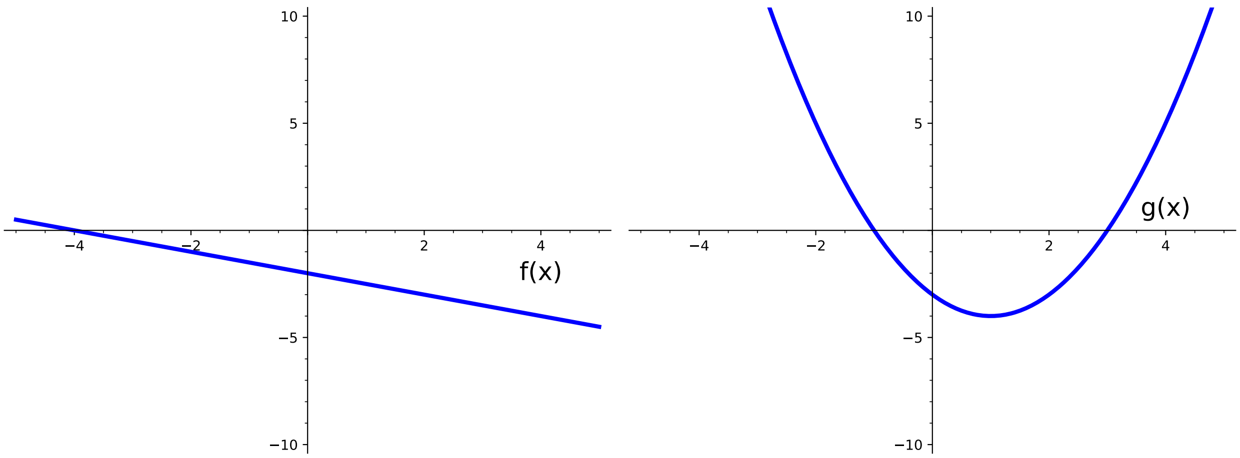
Objectives

- Use the graph of a function to find the domain and range in interval notation, the x- and y-intercepts, the maxima and minima, and where it is increasing and decreasing using interval notation.

Remark 2.3.1 In this section, we will be looking at different kinds of graphs and will identify various characteristics. These ideas can span all kinds of functions, so you will see these come up multiple times!

Definition 2.3.2 One of the easiest things to identify from a graph are the **intercepts**, which are points at which the graph crosses the axes. An x-**intercept** is a point at which the graph crosses the x-axis and a y-**intercept** is a point at which the graph crosses the y-axis. Because intercepts are points, they are typically written as an ordered pair: (x, y). $\diamond$

Activity 2.3.3 Use the following graphs to answer the questions.



(a) What are the x-intercept(s) of $f(x)$?

A. $(0, -4)$

B. $(-2, 0)$

C. $(-4, 0)$

D. $(0, -2)$

(b) What are the x-intercept(s) of $g(x)$?

A. $(0, -3)$

B. $(-1, 0)$

C. $(3, 0)$

D. $(-3, 0)$

(c) What are the y-intercept(s) of $f(x)$?

A. $(0, -4)$

B. $(-2, 0)$

C. $(-4, 0)$

D. $(0, -2)$

(d) What are the y-intercept(s) of $g(x)$?

A. $(0, -3)$

B. $(-1, 0)$

C. $(3, 0)$

D. $(-3, 0)$

(e) Sketch a graph of a function with the following intercepts:

- x-intercepts: $(-2, 0)$ and $(6, 0)$
- y-intercept: $(0, 4)$

(f) Sketch a graph of a function with the following intercepts:

Characteristics of a Function's Graph (FN3)

- x-intercept: $(-1, 0)$
- y-intercept: $(0, 6)$ and $(0, -2)$

Remark 2.3.4 Notice in Activity 2.3.3, that a function can have multiple x-intercepts, but only one y-intercept. Having more than one y-intercept would create a graph that is not a function!

Definition 2.3.5 The **domain** refers to the set of possible input values and the **range** refers to the set of possible output values. If given a graph, however, it would be impossible to list out all the values for the domain and range so we use interval notation to represent the set of values.

Recall that the terms **domain** and **range** were first introduced in Definition 2.1.1.

$\Diamond$

Activity 2.3.6 Use the following graph to answer the questions below.

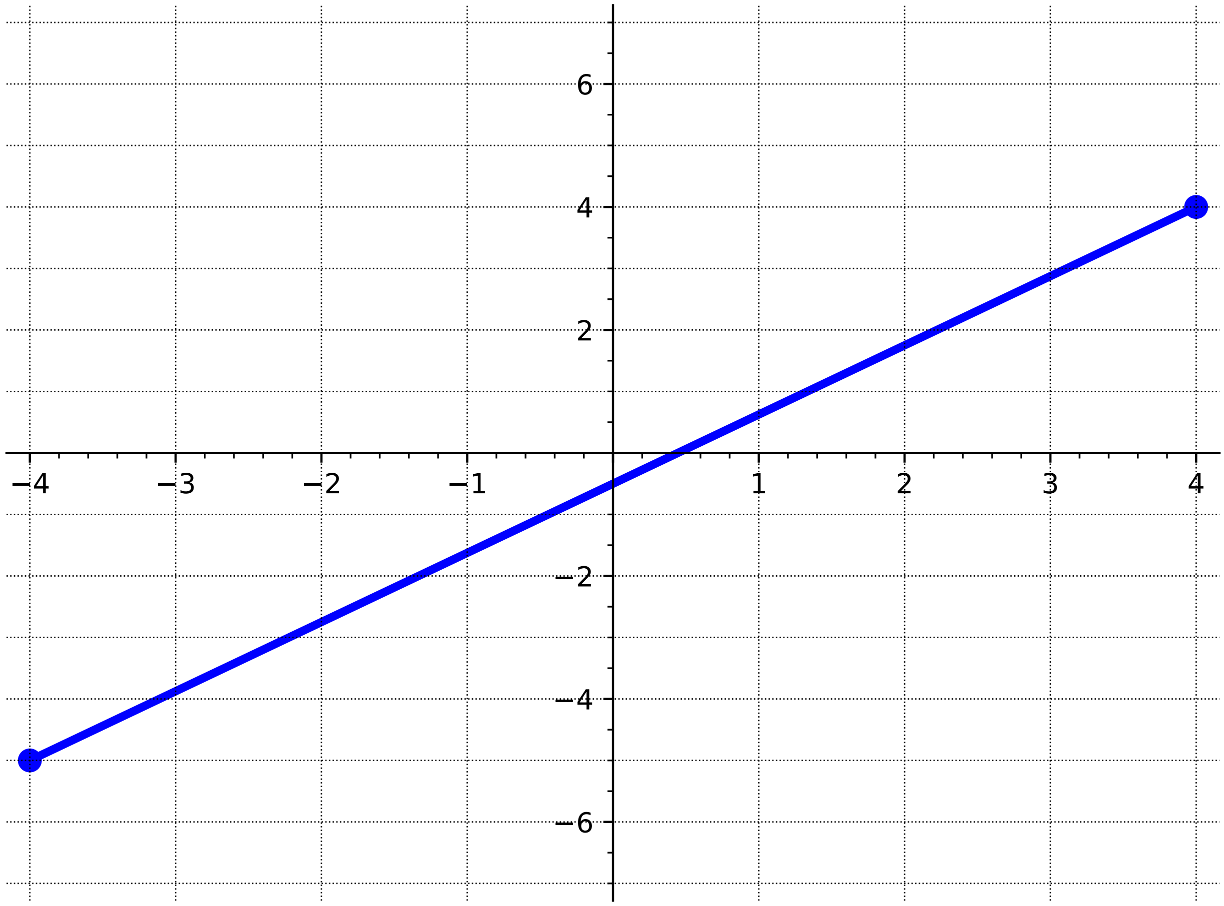


Figure 2.3.7

(a) Draw on the x-axis all the values in the domain.

(b) What interval represents the domain you drew in part (a)?

A. $[4, -4]$
B. $[-4, 4]$
C. $(-4, 4)$
D. $(4, -4)$

(c) Draw on the y-axis all the values in the range.

(d) What interval represents the range you drew in part (c)?

A. $(-5, 4)$
B. $[-4, 4]$
C. $[-5, 4]$
D. $(4, -5)$

Activity 2.3.8 Use the following graph to answer the questions below.

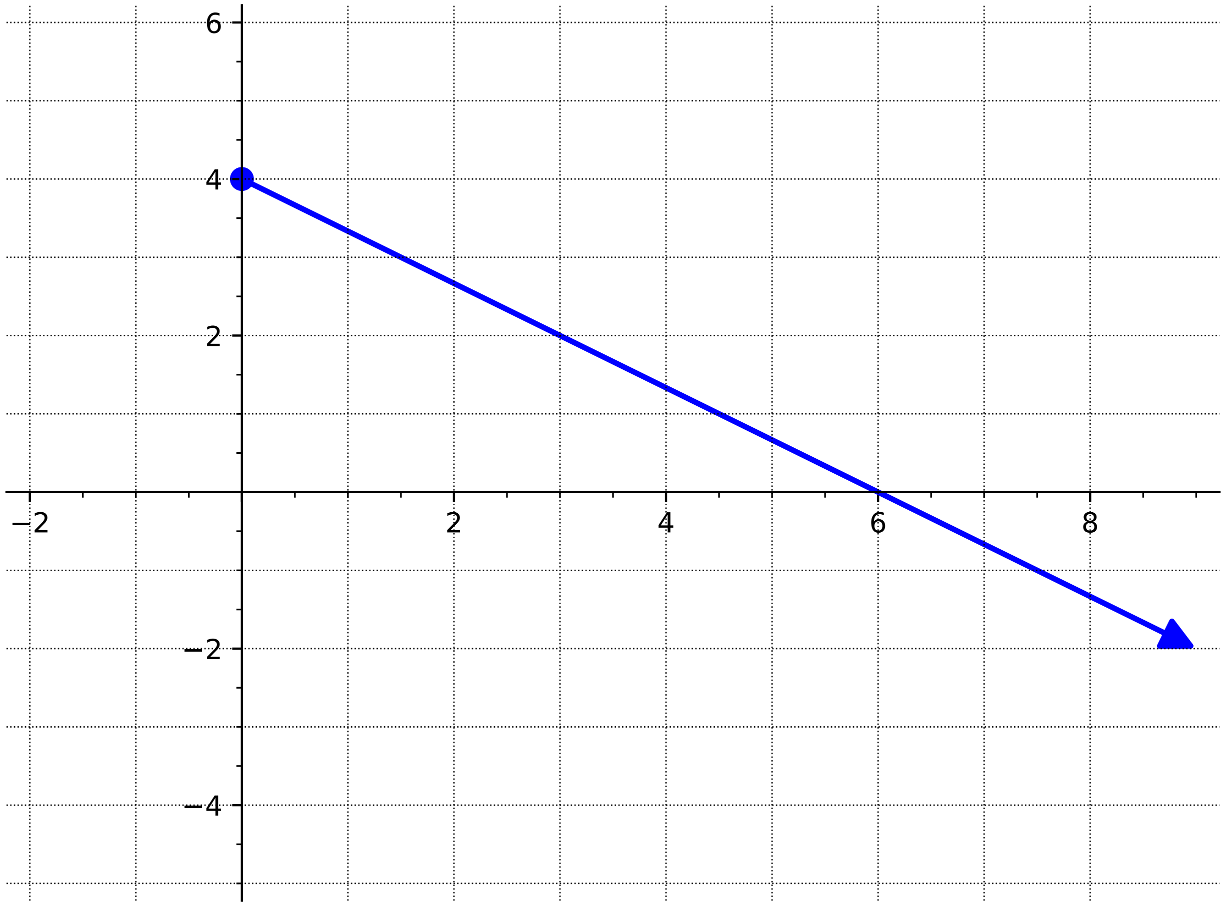


Figure 2.3.9

(a) What is the domain of this graph?

A. $[4, \infty)$

B. $(-\infty, 0]$

C. $(-\infty, 4]$

D. $[0, \infty)$

(b) What is the range of this graph?

A. $[4, \infty)$

B. $(-\infty, 0]$

C. $(-\infty, 4]$

D. $[0, \infty)$

Remark 2.3.10 When writing your intervals for domain and range, notice that you will need to write them from the smallest values to the highest values. For example, we wouldn't write $[4, -\infty)$ as an interval because $-\infty$ is smaller than 4. For domain, read the graph from left to right. For range, read the graph from bottom to top.

Activity 2.3.11 Use the following graph to answer the questions below.

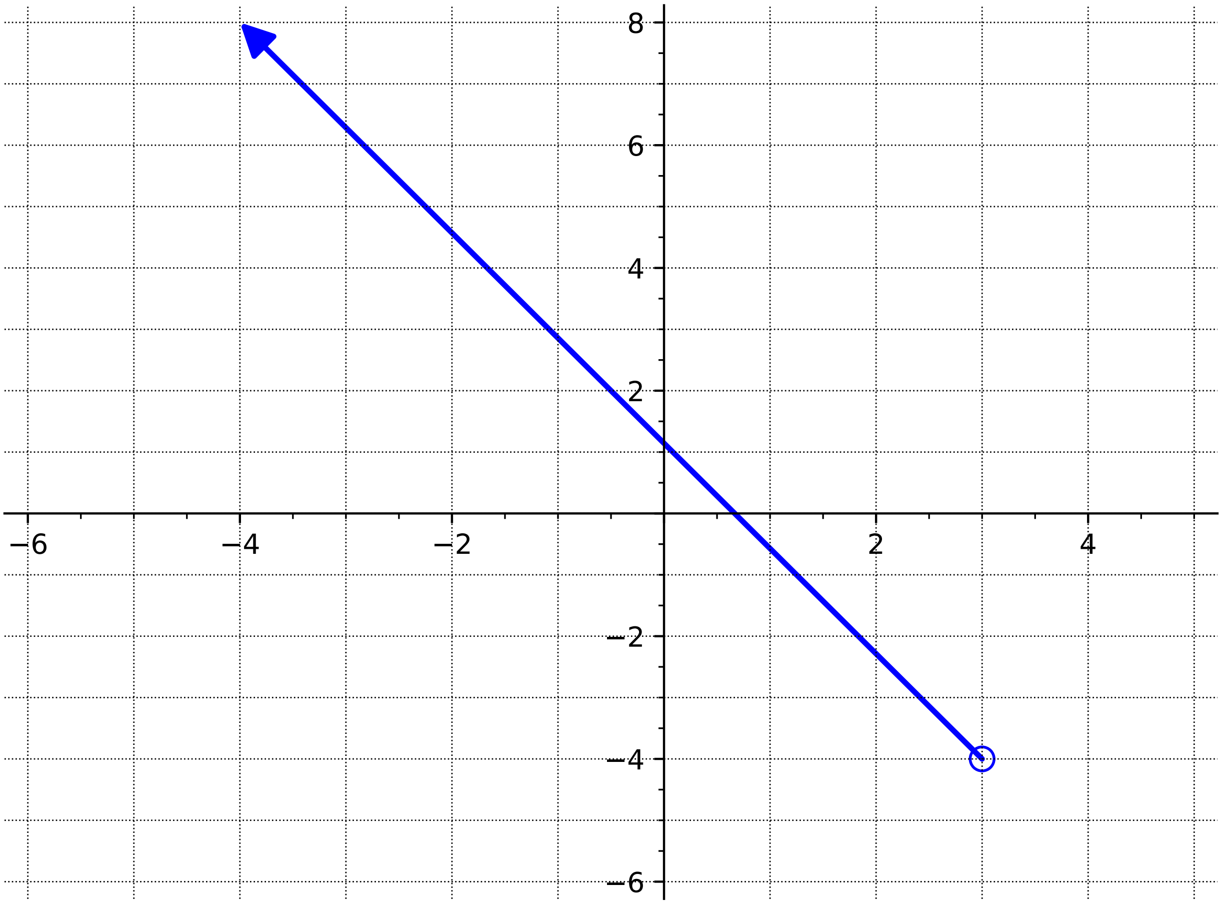


Figure 2.3.12

(a) What is the domain of this graph?

A. $(-\infty, 3)$

B. $(\infty, -4]$

C. $(-4, \infty)$

D. $(-\infty, 3]$

(b) What is the range of this graph?

A. $(-\infty, 3)$

B. $(\infty, -4]$

C. $(-4, \infty)$

D. $(-\infty, 3]$

Activity 2.3.13 Use the following graph to answer the questions below.

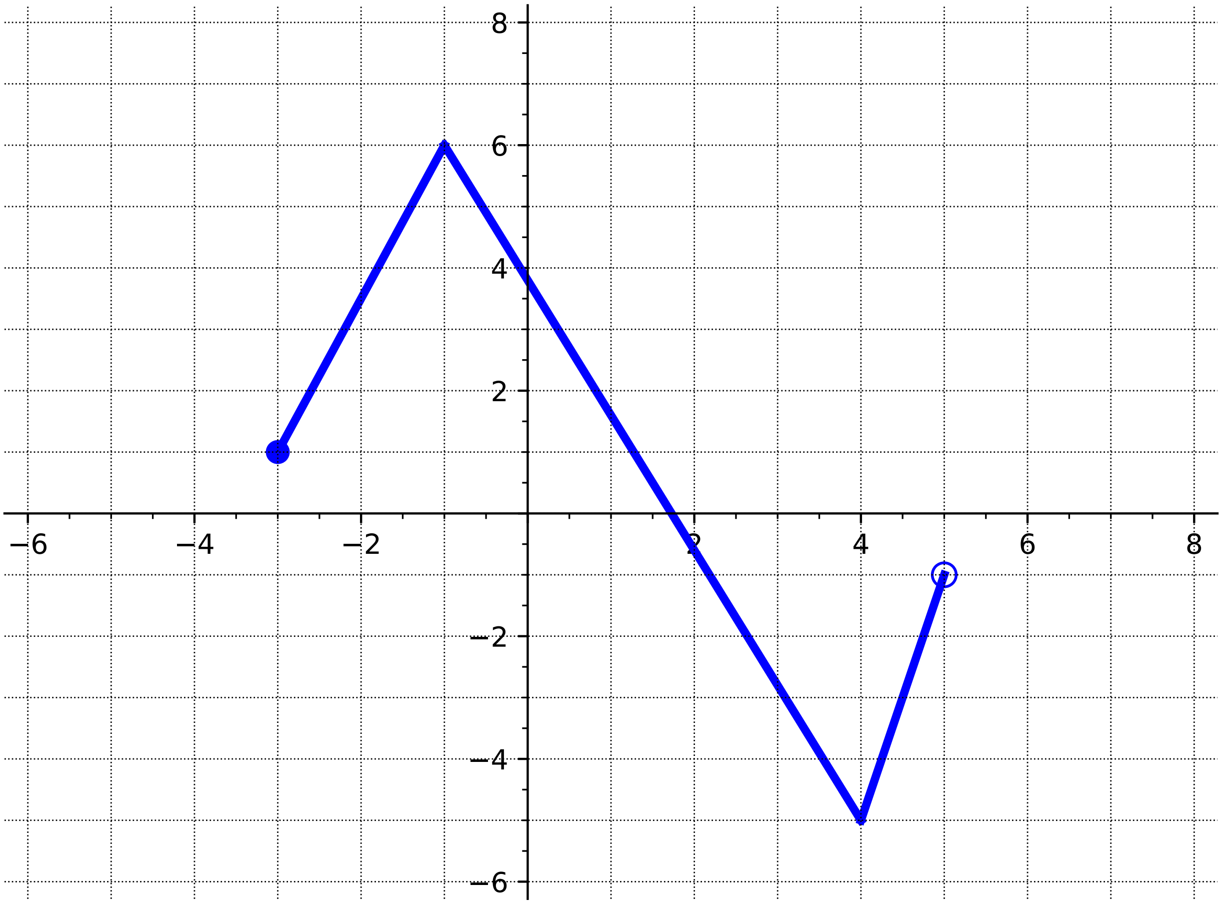


Figure 2.3.14

(a) What is the domain of this graph?

A. $(-3, 5)$

B. $(-5, 7)$

C. $[-5, 7]$

D. $[-3, 5)$

(b) What is the range of this graph?

A. $(-3, 5)$

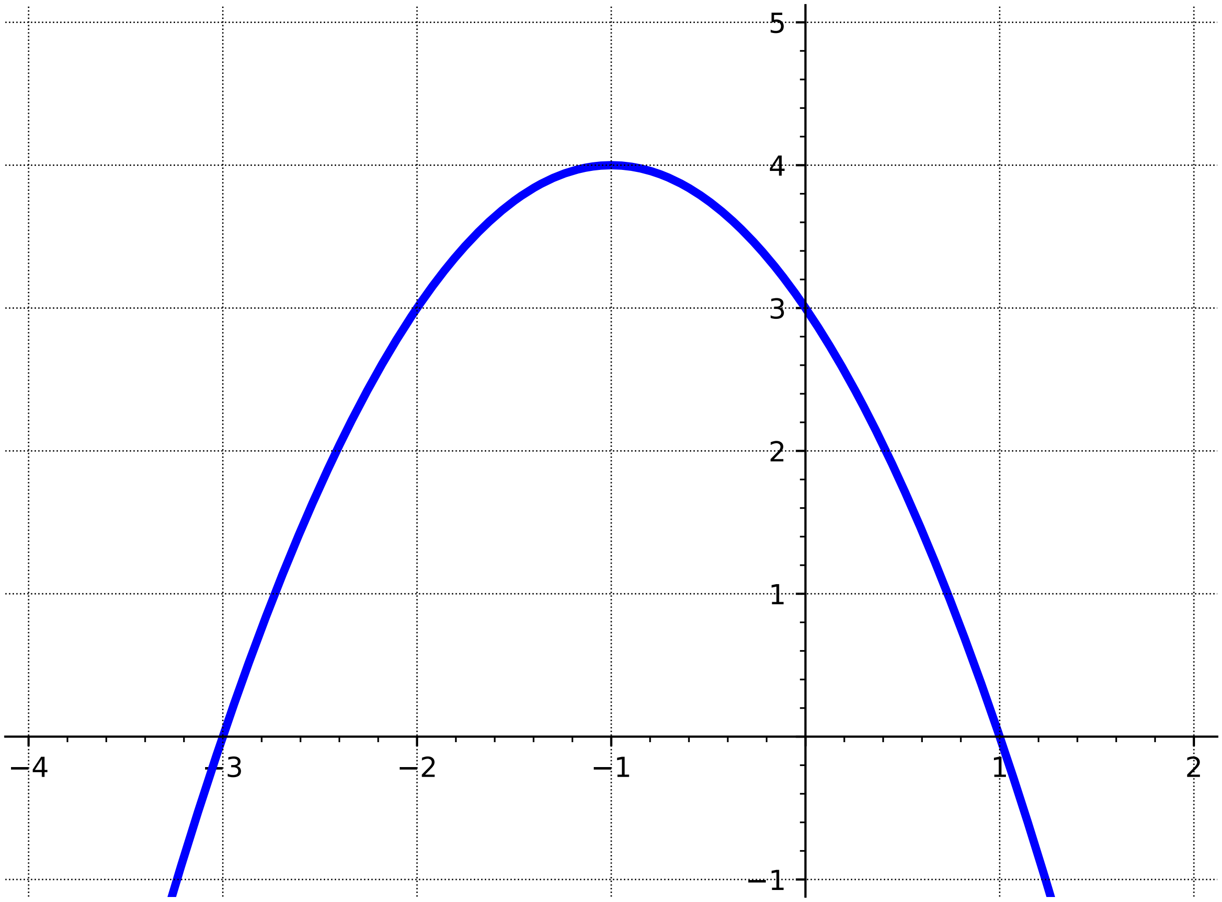
B. $(-5, 6)$

C. $[-5, 6]$

D. $[-3, 5)$

Remark 2.3.15 Notice that finding the domain and range can be tricky! Be sure to pay attention to the x- and y-values of the entire graph - not just the endpoints!

Activity 2.3.16 In this activity, we will look at where the function is increasing and decreasing. Use the following graph to answer the questions below.



(a) Where do you think the graph is increasing?

(b) Which interval best represents where the function is increasing?

A. $(-\infty, -1]$

B. $(-\infty, -1)$

C. $(-1, \infty)$

D. $[-1, \infty)$

(c) Where do you think the graph is decreasing?

(d) Which interval best represents where the function is decreasing?

A. $(-\infty, -1]$

B. $(-\infty, -1)$

C. $(-1, \infty)$

D. $[-1, \infty)$

(e) Based on what you see on the graph, do you think this graph has any maxima or minima?

Definition 2.3.17 As you noticed in Activity 2.3.16, functions can increase or decrease (or even remain constant!) for a period of time. The **interval of increase** is when the y-values of the function increase as the x-values increase. The **interval of decrease** is when the y-values of the function decrease as the x-values increase. The function is constant when the y-values remain constant as x-values increase (also known as the **constant interval**).

The easiest way to identify these intervals is to read the graph from left to right and look at what is happening to the y-values. $\Diamond$

Definition 2.3.18 The **maximum**, or **global maximum**, of a graph is the point where the y-coordinate has the largest value. The **minimum**, or **global minimum** is the point on the graph where the y-coordinate has the smallest value.

Graphs can also have **local maximums** and **local minimums**. A local maximum point is a point where the function value (i.e, y-value) is larger than all others in some neighborhood around the point. Similarly, a local minimum point is a point where the function value (i.e, y-value) is smaller than all others in some neighborhood around the point. $\Diamond$

Remark 2.3.19 Global extrema are sometimes referred to as absolute extrema, while local extrema are sometimes referred to as relative extrema.

Activity 2.3.20 Use the following graph to answer the questions below.

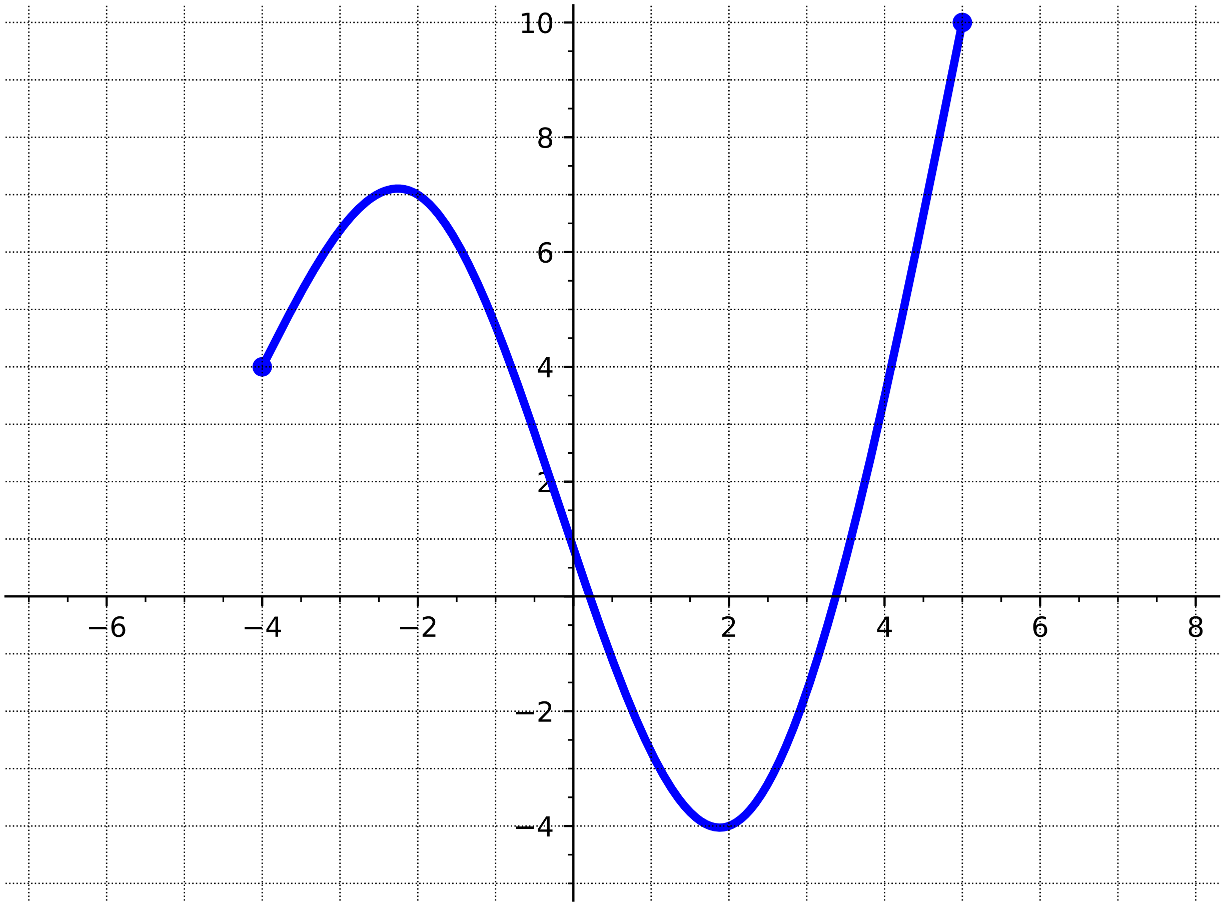


Figure 2.3.21

(a) At what value of x is there a global maximum?

A. $x = -4$

B. $x = -2$

C. $x = 2$

D. $x = 5$

(b) What is the global maximum value?

A. 10

B. 7

C. 4

D. -4

(c) At what value of x is there a global minimum?

A. $x = -4$

B. $x = -2$

C. $x = 2$

D. $x = 5$

(d) What is the global minimum value?

A. 10
B. 7
C. 4
D. -4

(e) At approximately what value of x is there a local maximum?

A. $x \approx -4$
B. $x \approx -2$
C. $x \approx 2$
D. $x \approx 5$

(f) What is the local maximum value?

A. 10
B. 7
C. 4
D. -4

(g) At approximately what value of x is there a local minimum?

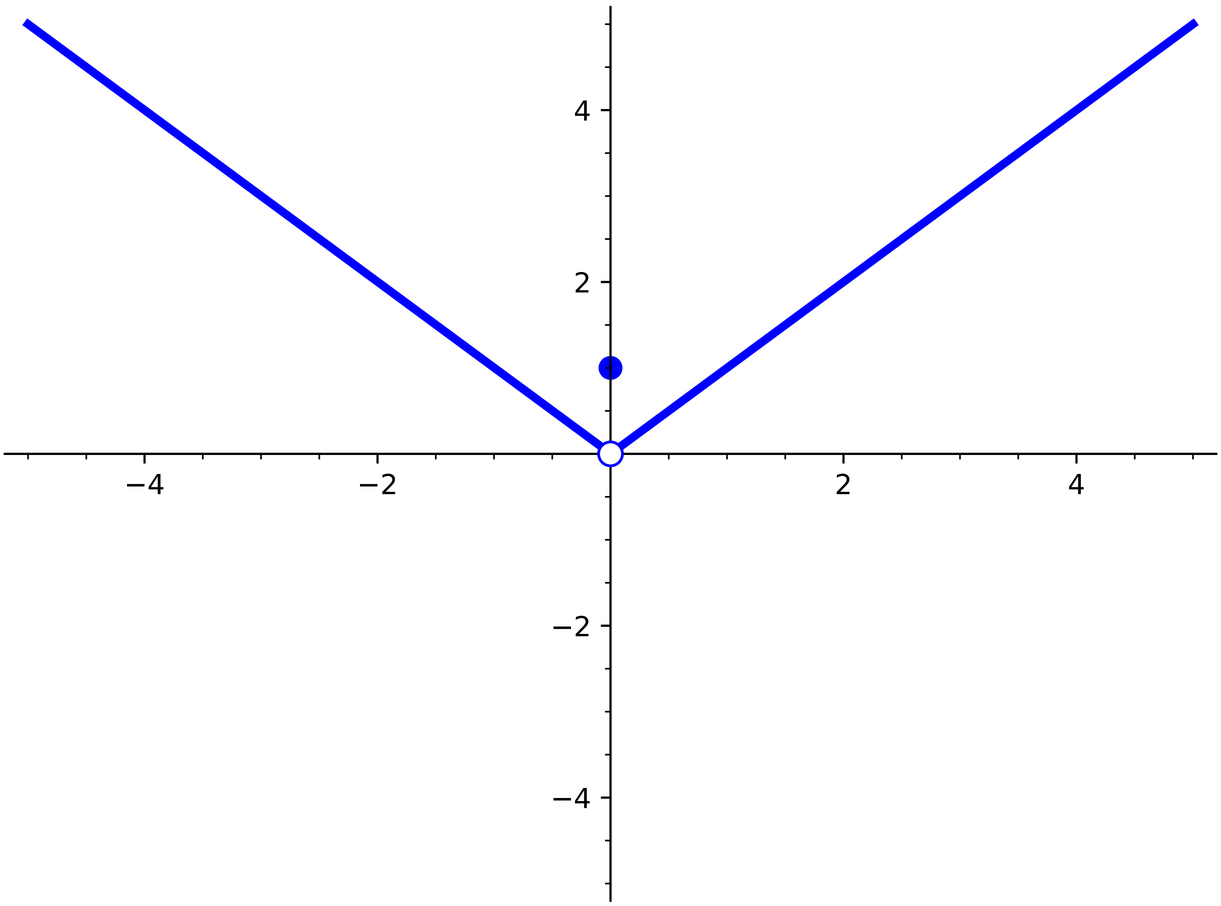
A. $x \approx -4$
B. $x \approx -2$
C. $x \approx 2$
D. $x \approx 5$

(h) What is the local minimum value?

A. 10
B. 7
C. 4
D. -4

Remark 2.3.22 Notice that in Activity 2.3.20, there are two ways we talk about max and min. We might want to know the location of where the max or min are (i.e., determining at which x-value the max or min occurs at) or we might want to know what the max or min values are (i.e., the y-value). Also, note that in Activity 2.3.20, a local minimum is also a global minimum.

Activity 2.3.23 Sometimes, it is not always clear what the maxima or minima are or if they exist. Consider the following graph of $f(x)$:



(a) What is the value of $f(0)$?

A. 1

B. 0

C. There is no local minimum

(b) What is the local minimum of $f(x)$?

A. 1

B. 0

C. There is no local minimum

(c) What is the global minimum of $f(x)$?

A. 1

B. 0

C. There is no global minimum

Activity 2.3.24 Use the following graph to answer the questions below.

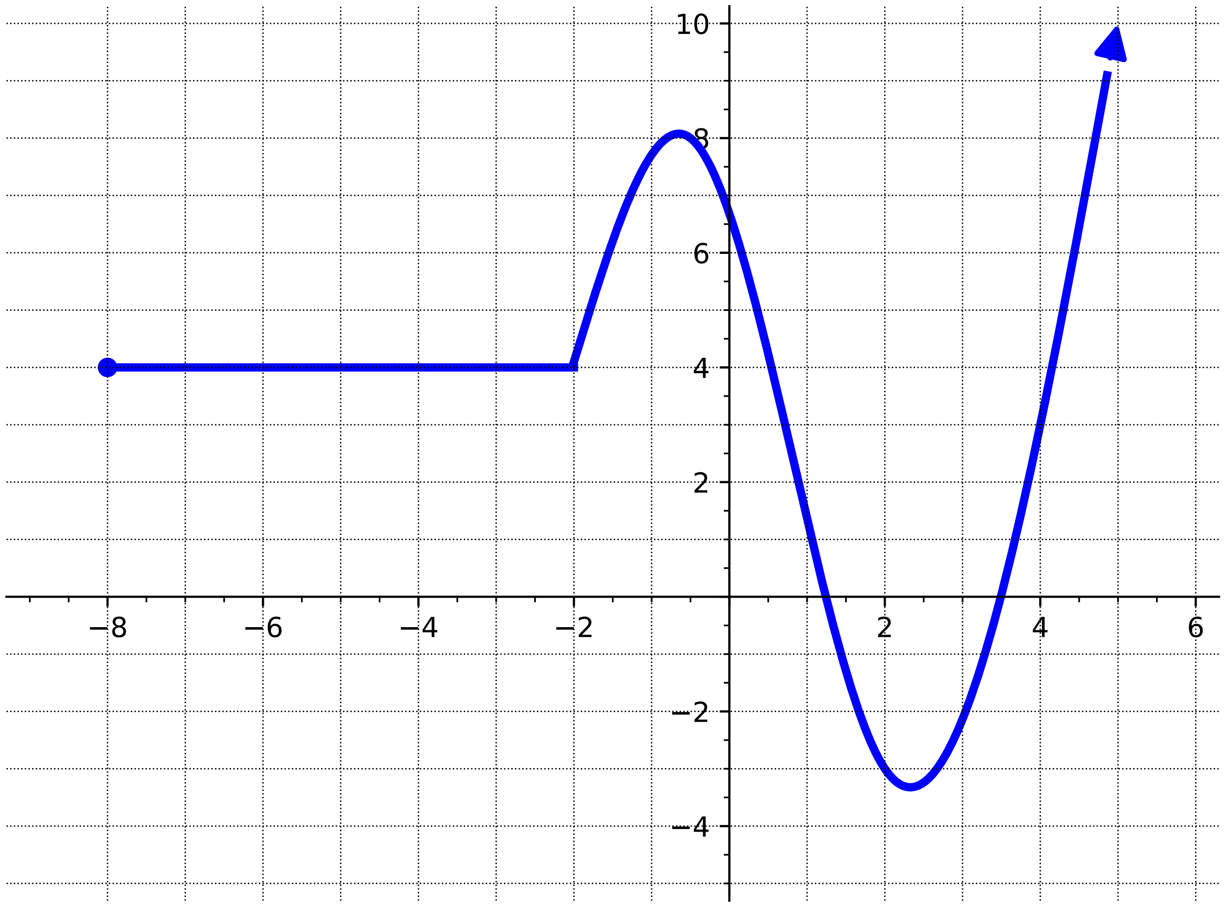


Figure 2.3.25

(a) What is the domain?

(b) What is the range?

(c) What is the x-intercept(s)?

(d) What is the y-intercept?

(e) Where is the function increasing?

(f) Where is the function decreasing?

(g) Where is the constant interval?

(h) At what x-values do the local maxima occur?

Characteristics of a Function's Graph (FN3)

(i) At what x-values do the local minima occur?

(j) What are the global max and min?

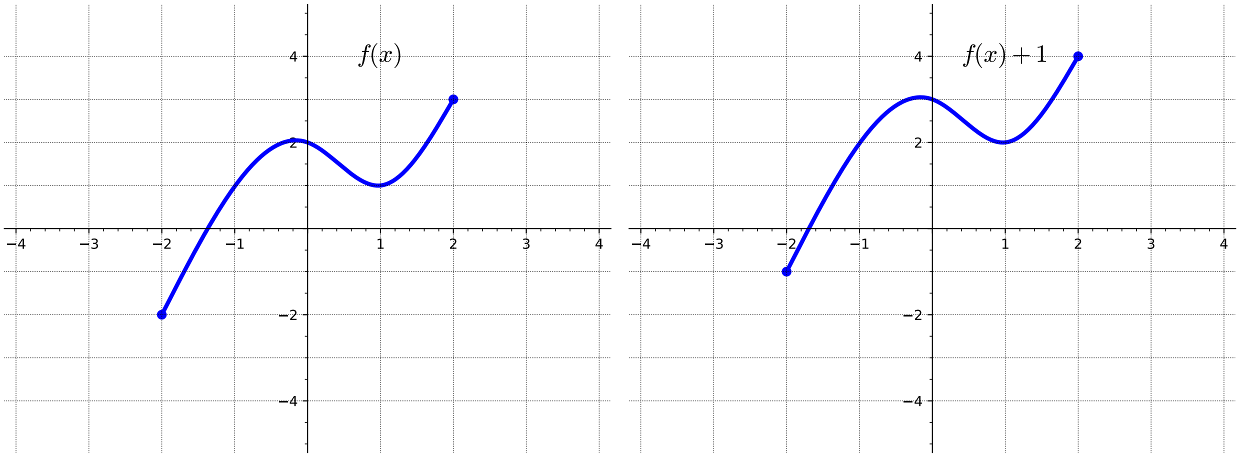
2.4 Transformation of Functions (FN4)

Objectives

- Apply transformations including horizontal and vertical shifts, stretches, and reflections to a function. Express the result of these transformations graphically and algebraically.

Remark 2.4.1 Informally, a transformation of a given function is an algebraic process by which we change the function to a related function that has the same fundamental shape, but may be shifted, reflected, and/or stretched in a systematic way.

Activity 2.4.2 Consider the following two graphs.



(a) How is the graph of $f(x) + 1$ related to that of $f(x)$?

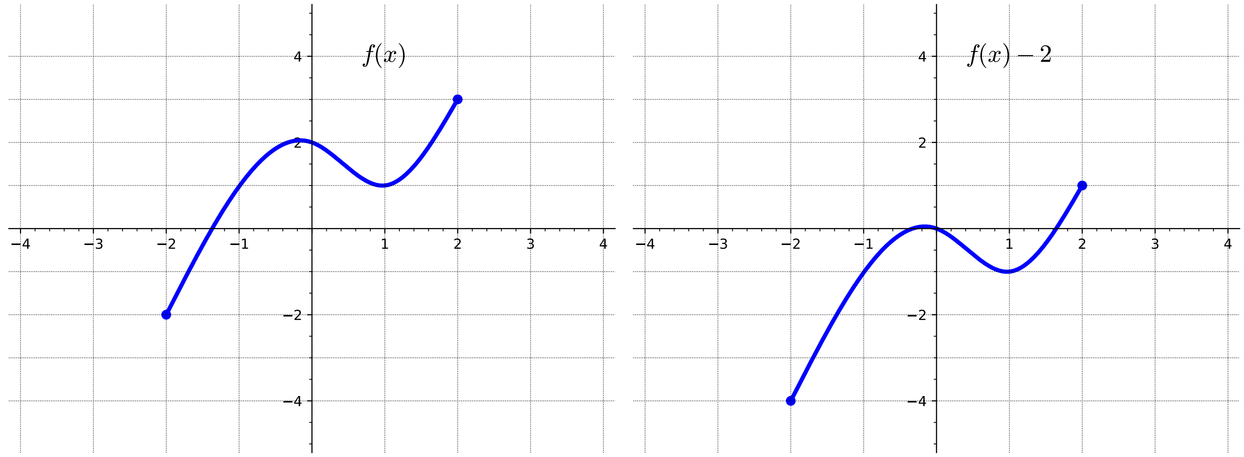
A. Shifted up 1 unit

B. Shifted left 1 unit

C. Shifted down 1 unit

D. Shifted right 1 unit

Activity 2.4.3 Consider the following two graphs.



(a) How is the graph of $f(x) - 2$ related to that of $f(x)$?

A. Shifted up 2 units

B. Shifted left 2 units

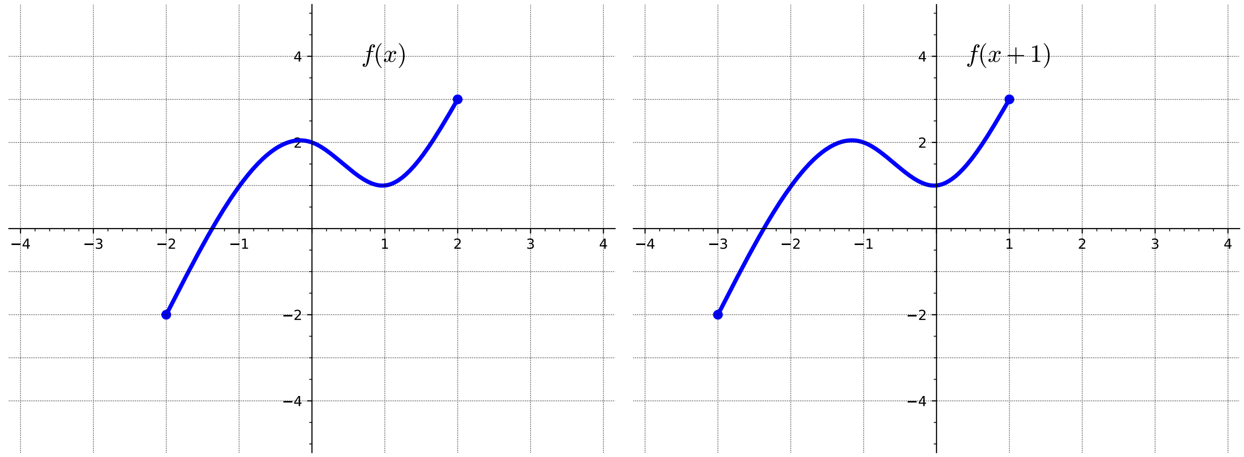
C. Shifted down 2 units

D. Shifted right 2 units

Remark 2.4.4 Notice that in Activity 2.4.2 and Activity 2.4.3, the y-values of the transformed graph are changed while the x-values remain the same.

Definition 2.4.5 Given a function $f(x)$ and a constant c, the transformed function $g(x) = f(x)+c$ is a **vertical translation** of the graph of $f(x)$. That is, all the outputs change by c units. If c is positive, the graph will shift up. If c is negative, the graph will shift down. $\Diamond$

Activity 2.4.6 Consider the following two graphs.



(a) How is the graph of $f(x+1)$ related to that of $f(x)$?

A. Shifted up by 1 unit

B. Shifted left 1 unit

C. Shifted down 1 unit

D. Shifted right 1 unit

Activity 2.4.7 Consider the following two graphs.

(a) How is the graph of $f(x-3)$ related to that of $f(x)$?

A. Shifted up by 3 units

B. Shifted left 3 units

C. Shifted down 3 units

D. Shifted right 3 units

Remark 2.4.8 Notice that in Activity 2.4.6 and Activity 2.4.7, the x-values of the transformed graph are changed while the y-values remain the same.

Definition 2.4.9 Given a function $f(x)$ and a constant c, the transformed function $g(x) = f(x + c)$ is a **horizontal translation** of the graph of $f(x)$. If c is positive, the graph will shift left. If c is negative, the graph will shift right. $\Diamond$

Activity 2.4.10 Describe how the graph of the function is a transformation of the graph of the original function f.

(a) $f(x-4)+1$

A. Shifted down 4 units

B. Shifted left 4 units

C. Shifted down 1 unit

D. Shifted right 4 units

E. Shifted up 1 unit

(b) $f(x+3)-2$

A. Shifted down 2 units

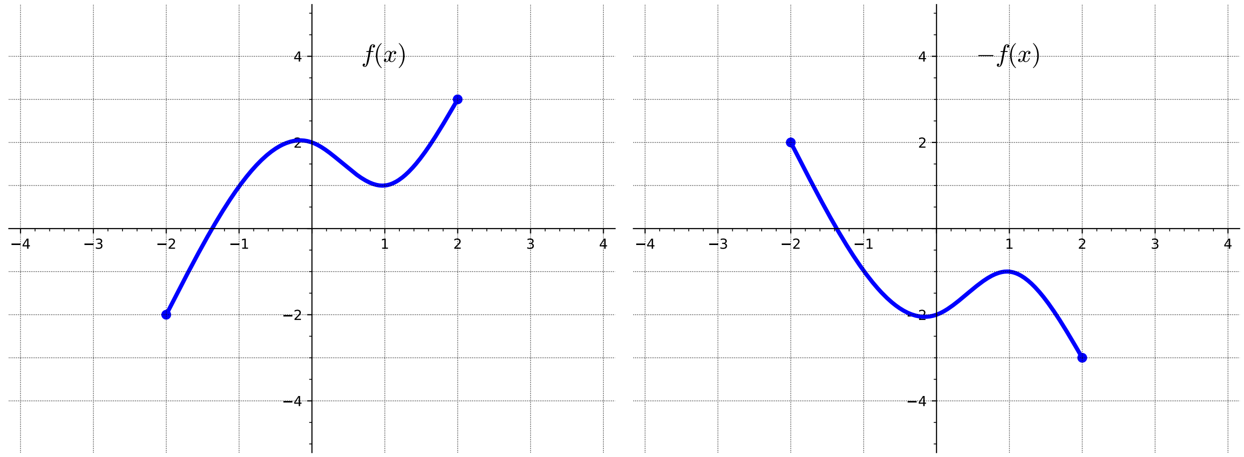
B. Shifted left 3 units

C. Shifted up 3 unit

D. Shifted right 3 units

E. Shifted up 2 unit

Activity 2.4.11 Consider the following two graphs.



(a) How is the graph of $-f(x)$ related to that of $f(x)$?

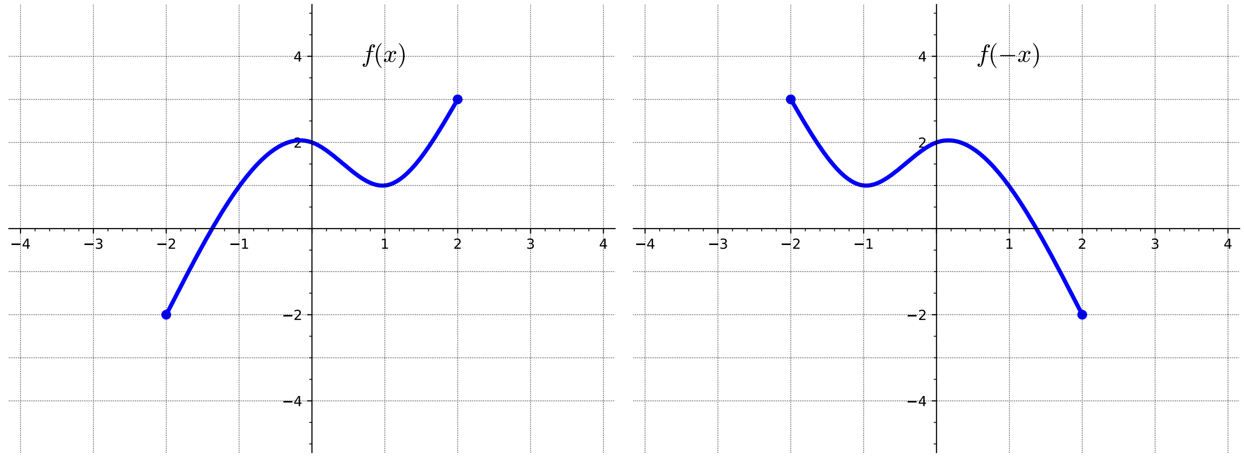
A. Shifted down 2 units

B. Reflected over the x-axis

C. Reflected over the y-axis

D. Shifted right 2 units

Activity 2.4.12 Consider the following two graphs.



(a) How is the graph of $f(-x)$ related to that of $f(x)$?

A. Shifted down 2 units

B. Reflected over the x-axis

C. Reflected over the y-axis

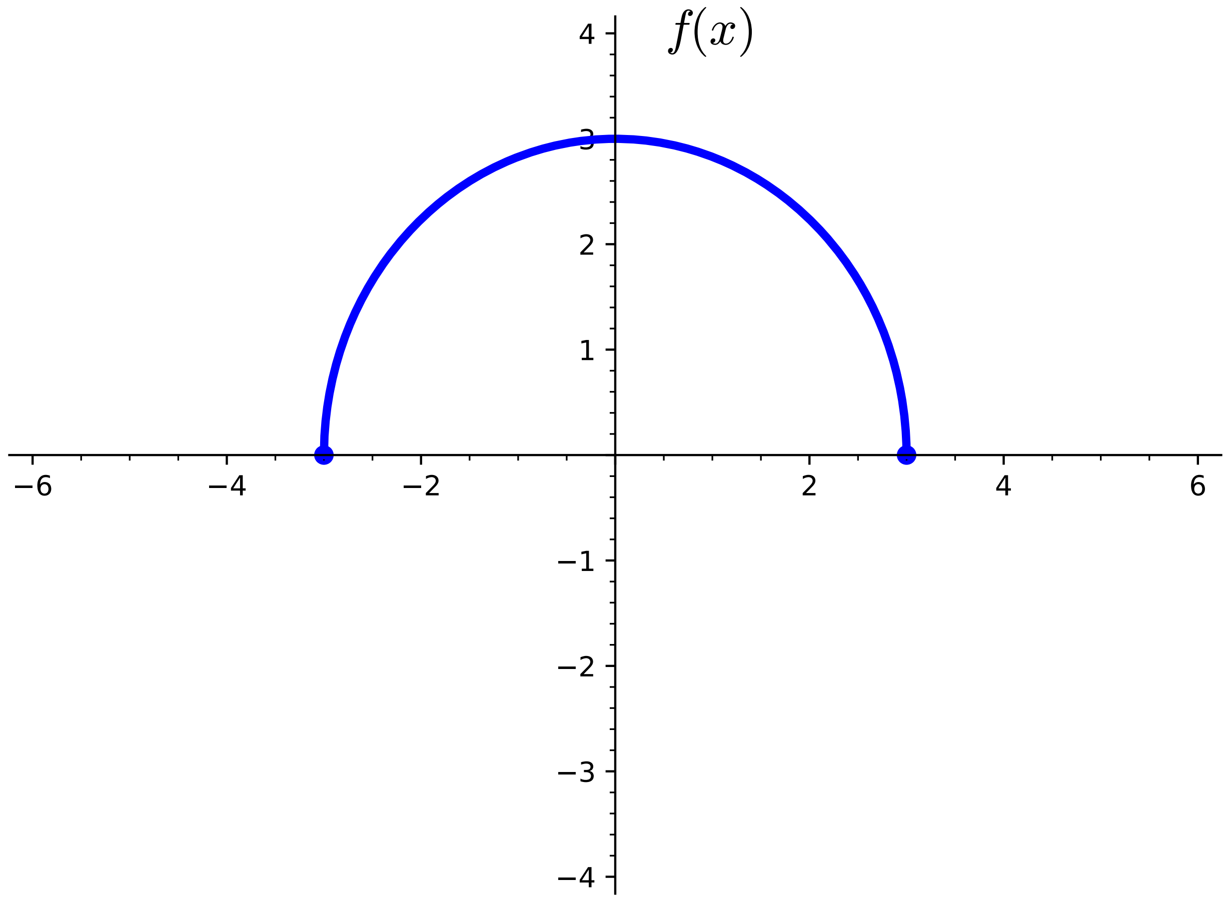
D. Shifted left 2 units

Remark 2.4.13 Notice that in Activity 2.4.11, the y-values of the transformed graph are changed while the x-values remain the same. While in Activity 2.4.12, the x-values of the transformed graph are changed while the y-values remain the same.

Definition 2.4.14 Given a function $f(x)$, the transformed function $g(x) = -f(x)$ is a **vertical reflection** of the graph of $f(x)$. That is, all the outputs are multiplied by -1. The new graph is a reflection of the old graph about the x-axis. $\Diamond$

Definition 2.4.15 Given a function $f(x)$, the transformed function $y = g(x) = f(-x)$ is a **horizontal reflection** of the graph of $f(x)$. That is, all the inputs are multiplied by -1. The new graph is a reflection of the old graph about the y-axis. $\Diamond$

Activity 2.4.16 Consider the following graph of the function $f(x)$.



(a) How is the graph of $-f(x+2)+3$ related to that of $f(x)$?

A. Shifted up 2 units

B. Shifted up 3 units

C. Reflected over the x-axis

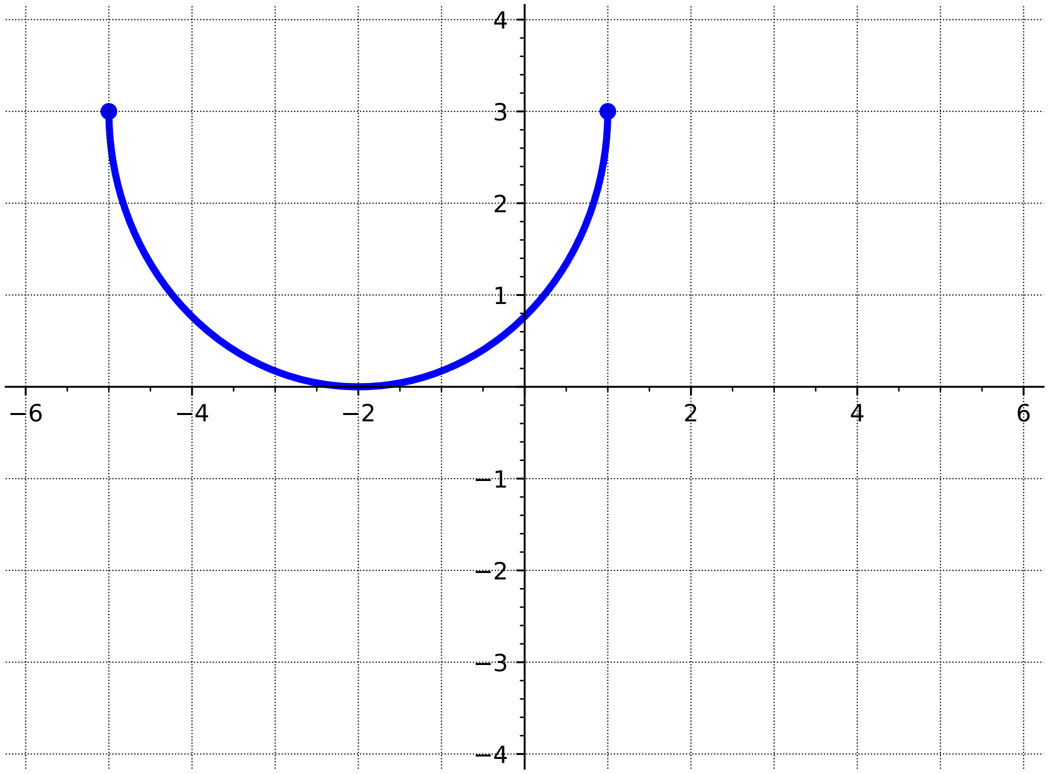
D. Reflected over the y-axis

E. Shifted left 3 units

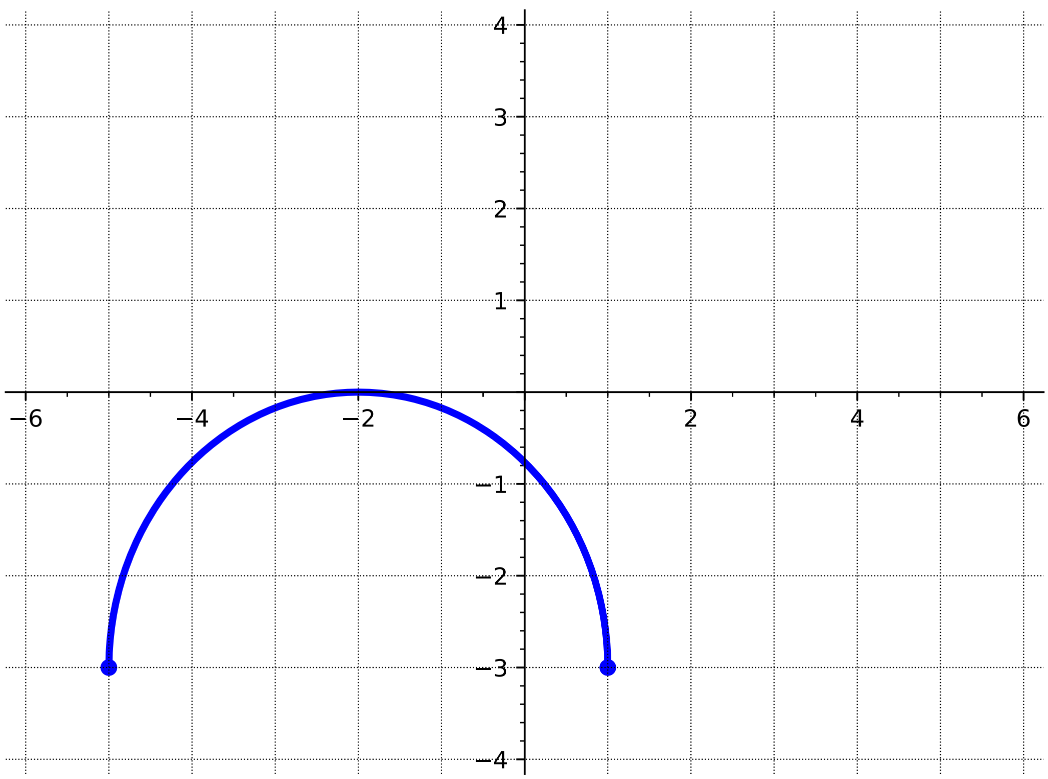
F. Shifted left 2 units

(b) Which of the following represents the graph of the transformed function $g(x) = -f(x+2)+3$?

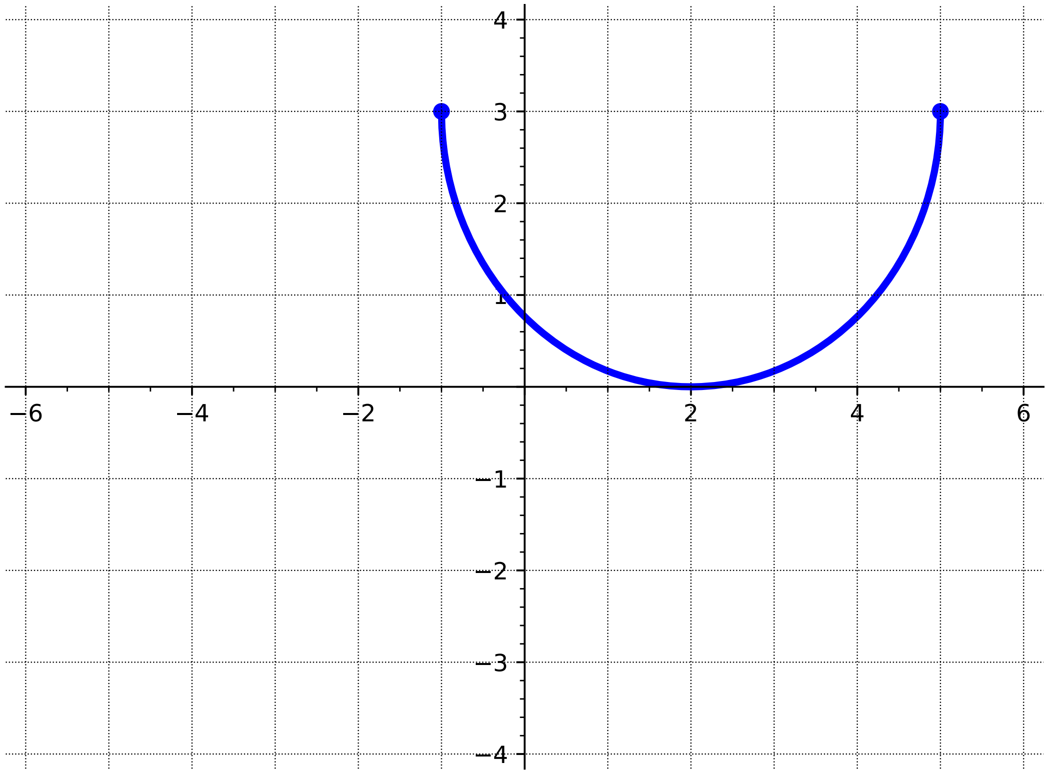
Transformation of Functions (FN4)



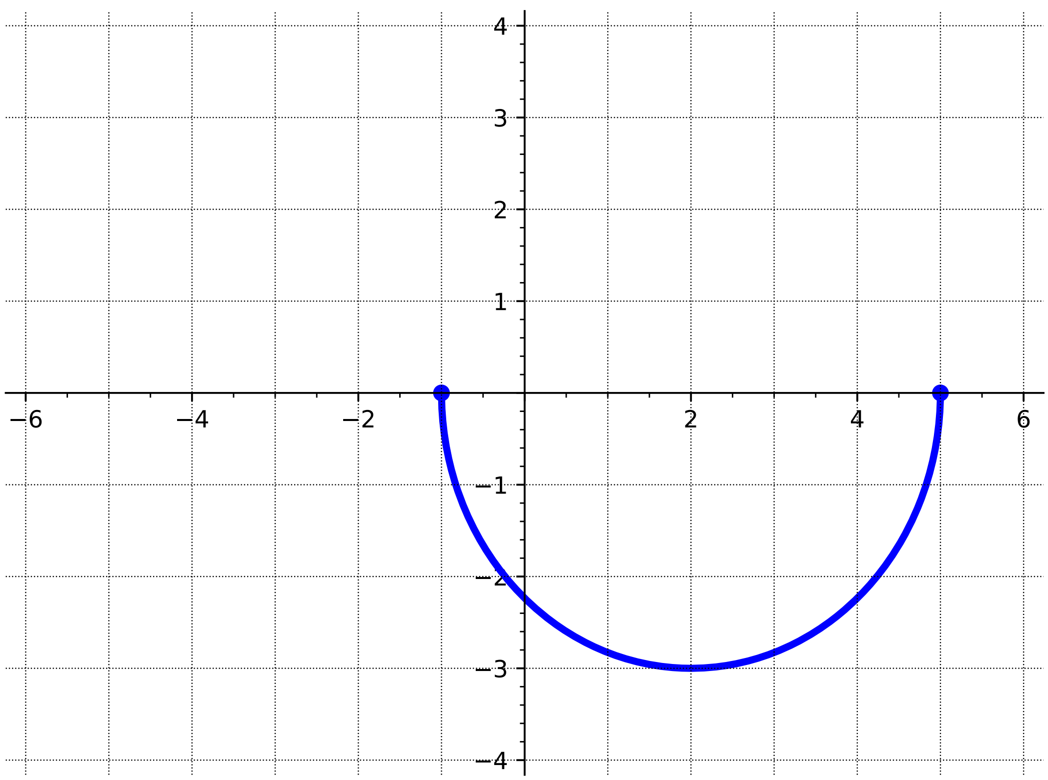
A.



B.



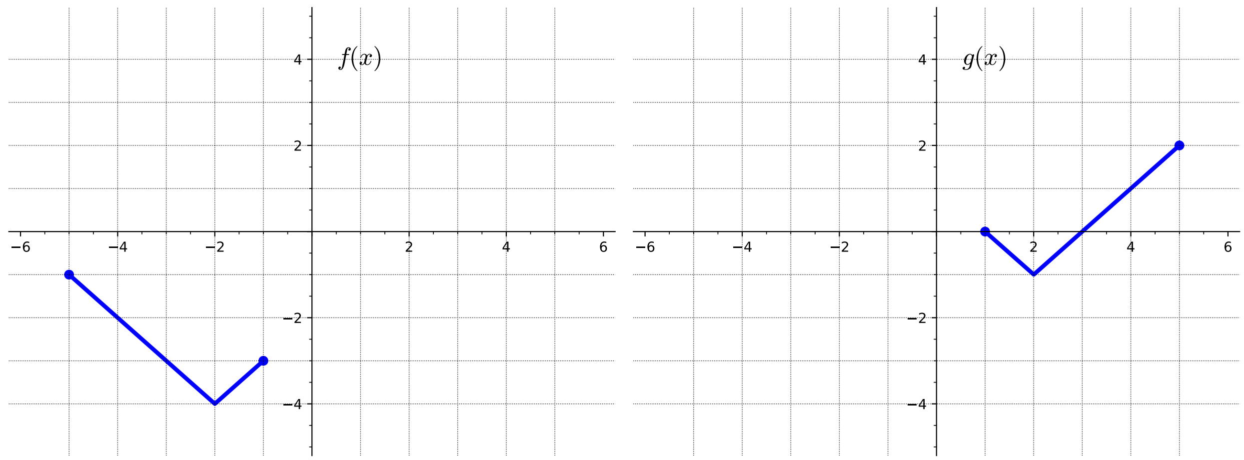
C.



D.

Remark 2.4.17 Notice that in Activity 2.4.16 the resulting graph is different if you perform the reflection first and then the vertical shift, versus the other order. When combining transformations, it is very important to consider the order of the transformations. Be sure to follow the order of operations.

Activity 2.4.18 Consider the following two graphs.



(a) How is the graph of $g(x)$ related to that of $f(x)$?

A. Shifted up 3 units

B. Shifted up 1 unit

C. Reflected over the x-axis

D. Reflected over the y-axis

E. Shifted left 1 unit

F. Shifted right 4 units

(b) List the order the transformations must be applied.

(c) Write an equation for the graphed function $g(x)$ using transformations of the graph $f(x)$.

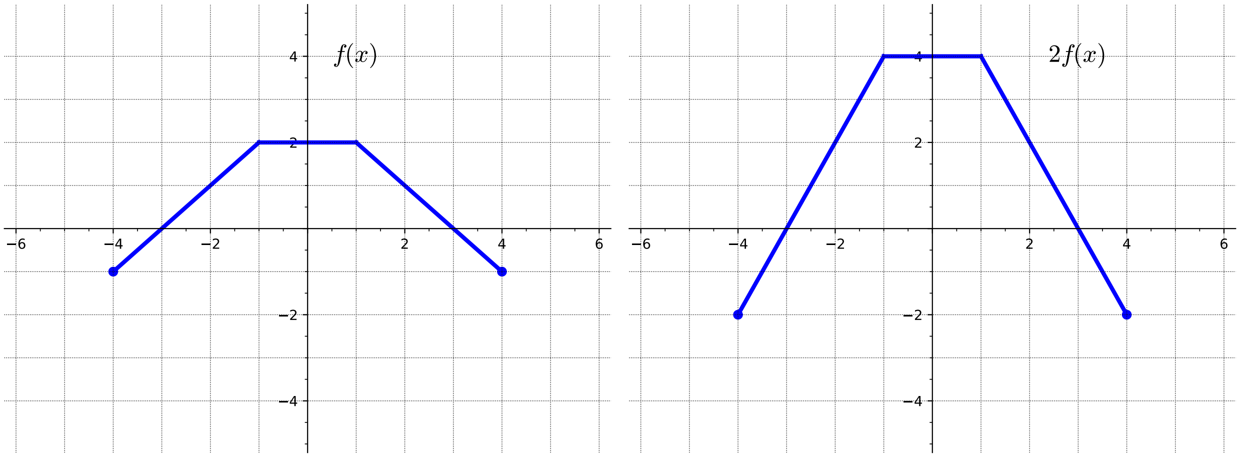
A. $g(x) = -f(x) + 3$

B. $g(x) = f(-x) + 3$

C. $g(x) = f(-x + 3)$

D. $g(x) = -f(x + 3)$

Activity 2.4.19 Consider the following two graphs.



(a) Consider the y-value of the two graphs at $x = 1$. How do they compare?

A. The y-value of $2f(x)$ is twice that of $f(x)$.

B. The y-value of $2f(x)$ is half that of $f(x)$.

C. The y-value of $2f(x)$ and $f(x)$ are the same.

D. The y-value of $2f(x)$ is negative that of $f(x)$.

(b) How is the graph of $2f(x)$ related to that of $f(x)$?

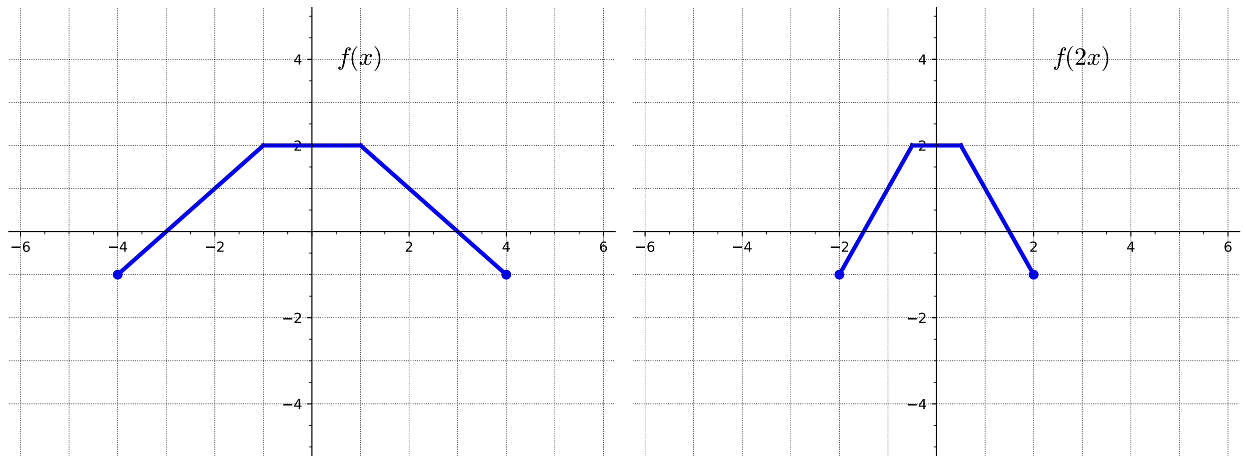
A. Vertically stretched by a factor of 2

B. Vertically compressed by a factor of 2

C. Horizontally stretched by a factor of 2

D. Horizontally compressed by a factor of 2

Activity 2.4.20 Consider the following two graphs.



(a) Consider a x-value of the two graphs at $y = 1$. How do they compare?

A. The x-value of $2f(x)$ is twice that of $f(x)$.

B. The x-value of $2f(x)$ is half that of $f(x)$.

C. The x-value of $2f(x)$ and $f(x)$ are the same.

D. The x-value of $2f(x)$ is negative that of $f(x)$.

(b) How is the graph of $f(2x)$ related to that of $f(x)$?

A. Vertically stretched by a factor of 2

B. Vertically compressed by a factor of 2

C. Horizontally stretched by a factor of 2

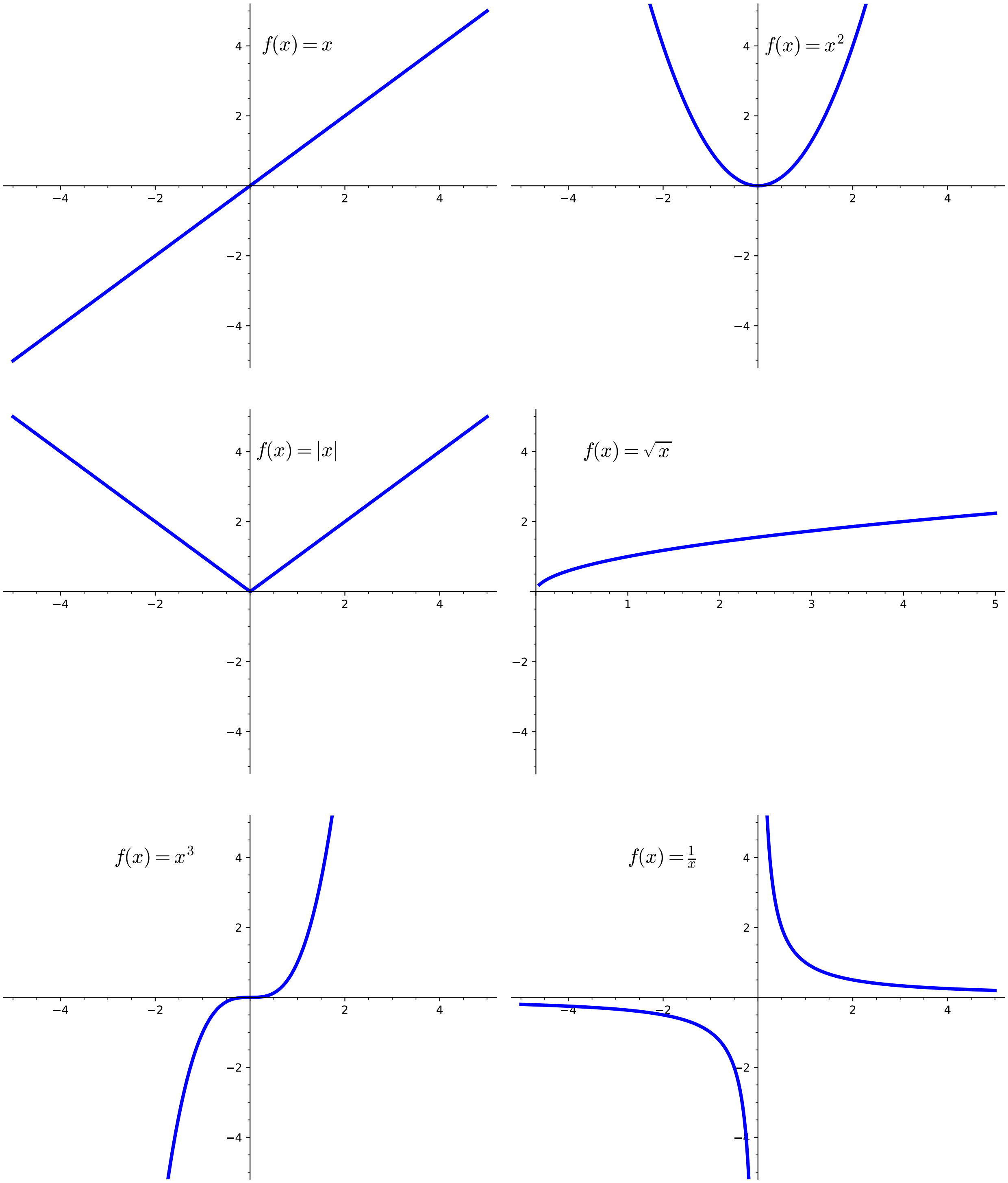
D. Horizontally compressed by a factor of 2

Remark 2.4.21 Notice that in Activity 2.4.19 the y-values are doubled while the x-values remain the same. While, in Activity 2.4.20 the x-values are cut in half while the y-values remain the same.

Definition 2.4.22 Given a function $f(x)$, the transformed function $g(x) = af(x)$ is a **vertical stretch** or **vertical compression** of the graph of $f(x)$. That is, all the outputs are multiplied by a. If $a > 1$, the new graph is a vertical stretch of the old graph away from the x-axis. If $0 < a < 1$, the new graph is a vertical compression of the old graph towards the x-axis. Points on the x-axis are unchanged. $\Diamond$

Definition 2.4.23 Given a function $f(x)$, the transformed function $g(x) = f(ax)$ is a **horizontal stretch** or **horizontal compression** of the graph of $f(x)$. That is, all the inputs are divided by a. If $a > 1$, the new graph is a horizontal compression of the old graph toward the y-axis. If $0 < a < 1$, the new graph is a horizontal stretch of the old graph away from the y-axis. Points on the y-axis are unchanged. $\diamondsuit$

Remark 2.4.24 We often use a set of basic functions with which to begin transformations. We call these parent functions.



Activity 2.4.25 Consider the function $g(x) = 3\sqrt{-x} + 2$

(a) Identify the parent function $f(x)$.

A. $f(x) = x^2$

B. $f(x) = |x|$

C. $f(x) = \sqrt{x}$

D. $f(x) = x$

(b) Graph the parent function $f(x)$.

(c) How is the graph of $g(x)$ related to that of the parent function$f(x)$?

A. Reflected over the x-axis

B. Reflected over the y-axis

C. Shifted down 2 units

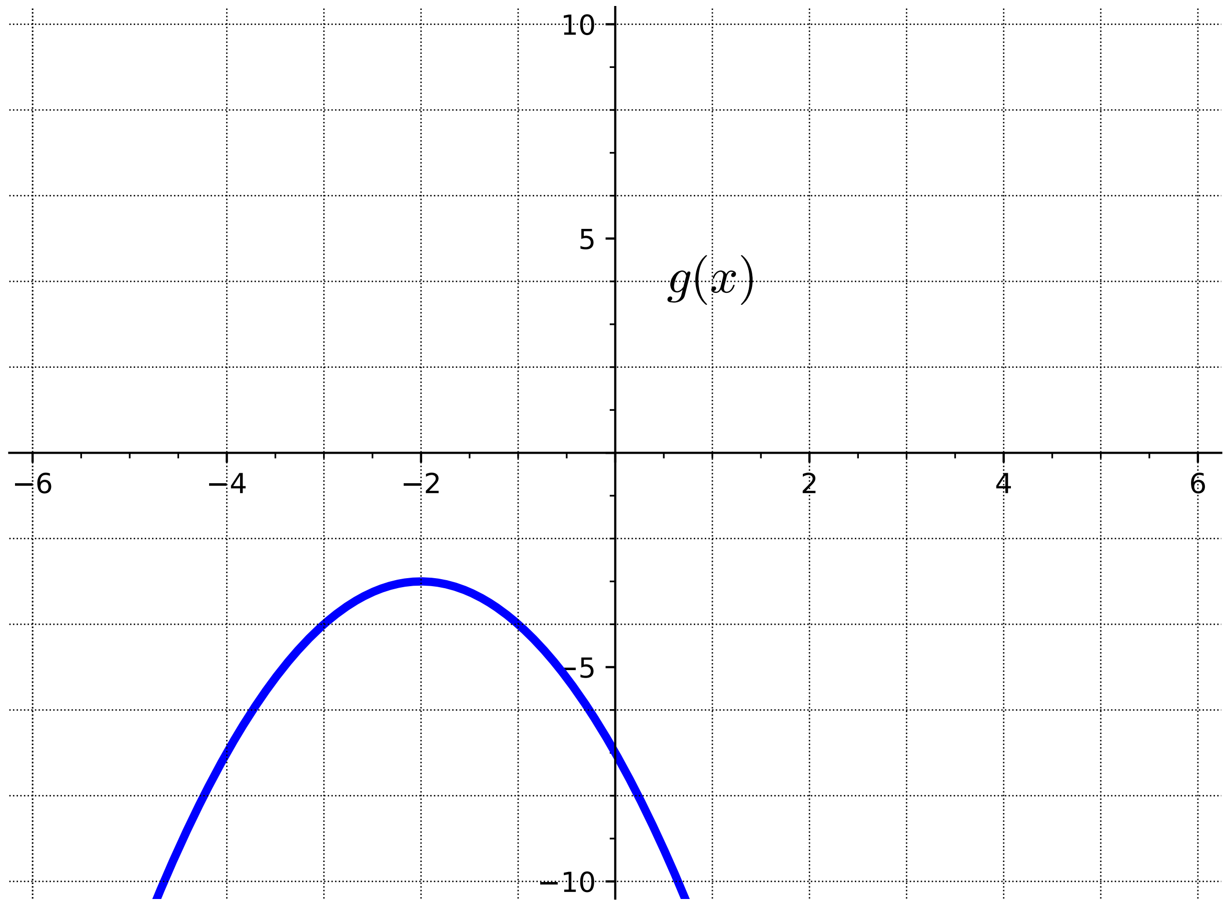
D. Shifted up 2 units

E. Vertically stretched by a factor of 3

F. Horizontally compressed by a factor of 3

(d) Graph the transformed function $g(x)$.

Activity 2.4.26 Consider the following graph of the function $g(x)$.



(a) Identify the parent function.

A. $f(x) = x^2$

B. $f(x) = |x|$

C. $f(x) = \sqrt{x}$

D. $f(x) = x$

(b) How is the graph of $g(x)$ related to that of the parent function$f(x)$?

A. Reflected over the x-axis

B. Reflected over the y-axis

C. Shifted down 3 units

D. Shifted up 3 units

E. Shifted left 2 units

F. Shifted right 2 units

(c) Write an equation to represent the transformed function $g(x)$.

A. $g(x) = -(x - 2)^2 - 3$

B. $g(x) = -(x + 2)^2 + 3$

C. $g(x) = (-x + 2)^2 - 3$

D. $g(x) = -(x + 2)^2 - 3$

2.5 Combining and Composing Functions (FN5)

Objectives

- Find the sum, difference, product, quotient, and composition of two or more functions and evaluate them.

Activity 2.5.1 Let $f(x) = x^2 - 3x$ and $g(x) = x^3 - 4x^2 + 7$.

(a) Which of the following seems likely to be the most simplified form of $f(x) + g(x)$?

A. $x^2 - 3x + x^3 - 4x^2 + 7$

B. $x^5 - 7x^3 + 7$

C. $-x^3 + 5x^2 - 3x - 7$

D. $x^3 - 3x^2 - 3x + 7$

(b) Which of the following seems likely to be the most simplified form of $f(x) - g(x)$?

A. $x^3 - 3x^2 - 3x + 7$

B. $-x^3 + 5x^2 - 3x - 7$

C. $-x^3 - 3x^2 - 3x + 7$

D. $x^2 - 3x - x^3 + 4x^2 - 7$

Activity 2.5.2 Let $f(x) = \sqrt{x+1}$ and $g(x) = 5x$.

(a) Which of the following seems likely to be the most simplified form of $f(x) \cdot g(x)$?

A. $\sqrt{5x+1}$

B. $5\sqrt{x+1}$

C. $\sqrt{5x^2+5x}$

D. $5x\sqrt{x+1}$

(b) Which of the following seems likely to be the most simplified form of $\frac{f(x)}{g(x)}$?

A. $\dfrac{5x}{\sqrt{x+1}}$

B. $\dfrac{\sqrt{x+1}}{5x}$

C. $\sqrt{\dfrac{x}{5x} + \dfrac{1}{5x}}$

D. $\sqrt{\dfrac{5x}{x} + \dfrac{5x}{1}}$

Remark 2.5.3 In Activity 2.5.1 and Activity 2.5.2, we have found the sum, difference, product, and quotient of two functions. We can use the following notation for these newly created functions:

$$
\begin{aligned}
(f+g)(x) &= f(x)+g(x) \\
(f-g)(x) &= f(x)-g(x) \\
(f\cdot g)(x) &= f(x)\cdot g(x) \\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}
\end{aligned}
$$

With $\left(\frac{f}{g}\right)(x)$, we note that the quotient is only defined when $g(x) \neq 0$.

Activity 2.5.4 Let $f(x) = \dfrac{1}{3x-5}$.

(a) Find $f(4)$.

A. $\dfrac{4}{3x-5}$

B. $\dfrac{1}{4(3x-5)}$

C. $\dfrac{1}{7}$

D. 7

Hint. See Remark 2.2.5 for a reminder of what this notation means!

(b) If you were asked to find $f(x^3-2)$, how do you think you would proceed?

A. Multiply the original function $\frac{1}{3x-5}$ by x^3-2.

B. Plug the expression x^3-2 in for all the x-values in $\frac{1}{3x-5}$.

C. Plug the original function $\frac{1}{3x-5}$ in for all the x-values in x^3-2.

D. Multiply $3x-5$ by x^3-2.

(c) Find $f(x^3-2)$.

A. $\dfrac{1}{3x-5} \cdot (x^3-2)$

B. $\dfrac{1}{3(x^3-2)-5}$

C. $\left(\dfrac{1}{3x-5}\right)^3 - 2$

D. $(3x-5)(x^3-2)$

(d) What if we gave the expression x^3-2 a name? Let's define $g(x) = x^3-2$. What's another way we could denote $f(x^3-2)$?

A. $f(x) \cdot g(x)$

B. $g(f(x))$

C. $f(g(x))$

D. $\dfrac{f(x)}{g(x)}$

Definition 2.5.5 Given the functions $f(x)$ and $g(x)$, we define the **composition of f and g** to be the new function $h(x)$ given by

$$h(x) = f(g(x)).$$

We also sometimes use the notation

$$f \circ g$$

or

$$(f \circ g)(x)$$

to refer to $f(g(x))$. $\Diamond$

Remark 2.5.6 When discussing the composite function $f(g(x))$, also written as $(f \circ g)(x)$, we often call $g(x)$ the "inner function" and $f(x)$ the "outer function". It is important to note that the inner function is actually the first function that gets applied to a given input, and then the outer function is applied to the output of the inner function.

Activity 2.5.7 Let $f(x) = \dfrac{1}{3x-5}$ and $g(x) = x^3 - 2$.

(a) Find $f(g(x))$.

A. $\dfrac{x^3-2}{3x-5}$

B. $\dfrac{1}{(3x-5)(x^3-2)}$

C. $\dfrac{1}{3(x^3-2)-5}$

D. $\left(\dfrac{1}{3x-5}\right)^3 - 2$

(b) Find $g(f(x))$.

A. $\dfrac{x^3-2}{3x-5}$

B. $\dfrac{1}{(3x-5)(x^3-2)}$

C. $\dfrac{1}{3(x^3-2)-5}$

D. $\left(\dfrac{1}{3x-5}\right)^3 - 2$

Remark 2.5.8 We can also evaluate the composition of two functions at a particular value just as we did with one function. For example, we may be asked to find something like $f(g(2))$ or $(g \circ f)(-3)$.

Activity 2.5.9 Let $f(x) = 2x^3$ and $g(x) = \sqrt{6-x}$.

(a) Find $f(g(2))$.

A. 14

B. 16

C. 18

D. 20

E. undefined

(b) Find $(g \circ f)(-3)$.

A. 50

B. 54

C. $\sqrt{60}$

D. $\sqrt{-48}$

E. undefined

(c) Find $(f \circ g)(10)$.

A. $2(\sqrt{-4})^3$

B. 16

C. $\sqrt{-1994}$

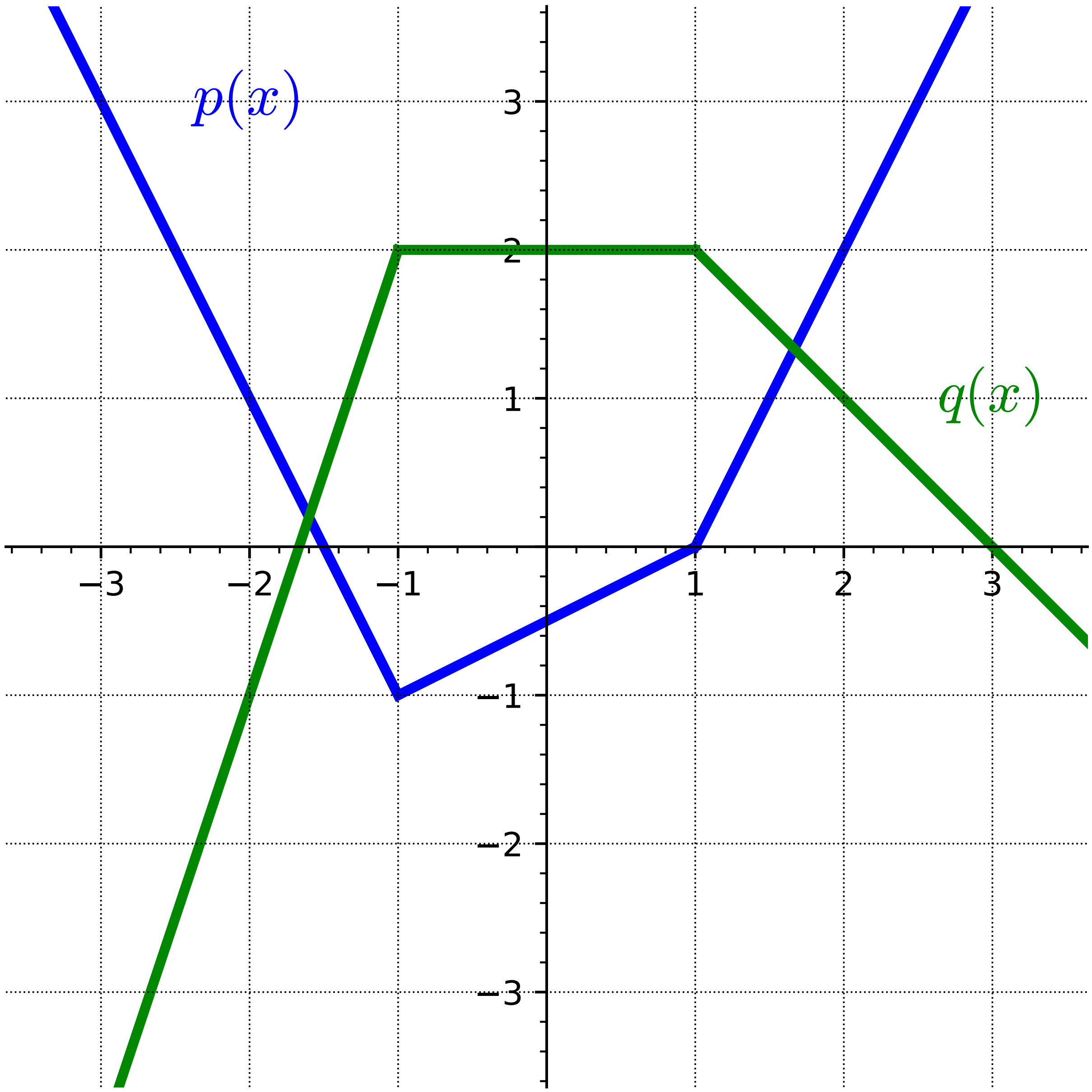
D. -16

E. undefined

Remark 2.5.10 As we saw in Activity 2.5.9, in order for a composite function to make sense, we need to ensure that the range of the inner function lies within the domain of the outer function so that the resulting composite function is defined at every possible input.

Remark 2.5.11 In addition to the possibility that functions are given by formulas, functions can be given by tables or graphs. We can think about composite functions in these settings as well, and the following activities prompt us to consider functions given in this way.

Activity 2.5.12 Let functions p and q be given by the graphs below.



Find each of the following. If something is not defined, explain why.

(a) $(p \circ q)(0)$

(b) $q(p(0))$

(c) $p(p(1))$

(d) $(q \circ p)(-3)$

(e) Find two values of x such that $q(p(x)) = 2$.

Activity 2.5.13 Let functions f and g be given by the tables below.

x	$f(x)$
0	6
1	4
2	3
3	4
4	7

Table 2.5.14

x	$g(x)$
0	1
1	3
2	0
3	5
4	2

Table 2.5.15

Find each of the following. If something is not defined, explain why.

(a) $(f \circ g)(2)$

(b) $(g \circ f)(3)$

(c) $g(f(4))$

(d) For what value(s) of x is $f(g(x)) = 4$?

(e) What are the domain and range of $(f \circ g)(x)$?

2.6 Finding the Inverse Function (FN6)

Objectives

- Find the inverse of a one-to-one function.

Finding the Inverse Function (FN6)

Remark 2.6.1 A function is a process that converts a collection of inputs to a corresponding collection of outputs. One question we can ask is: for a particular function, can we reverse the process and think of the original function's outputs as the inputs?

Activity 2.6.2 Temperature can be measured using many different units such as Fahrenheit, Celsius, and Kelvin. Fahrenheit is what is usually reported on the news each night in the United States, while Celsius is commonly used for scientific work. We will begin by converting between these two units. To convert from degrees Fahrenheit to Celsius use the following formula.

$$C = \frac{5}{9}(F - 32)$$

(a) Room temperature is around 68 degrees Fahrenheit. Use the above equation to convert this temperature to Celsius.

A. 5.8
B. 20
C. 155.4
D. 293

(b) Solve the equation $C = \frac{5}{9}(F - 32)$ for F in terms of C.

A. $F = \dfrac{5}{9}C + 32$
B. $F = \dfrac{5}{9}C - 32$
C. $F = \dfrac{9}{5}(C + 32)$
D. $F = \dfrac{9}{5}C + 32$

(c) Alternatively, 20 degrees Celsius is a fairly comfortable temperature. Use your solution for F in terms of C to convert this temperature to Fahrenheit.

A. 43.1
B. -20.9
C. 93.6
D. 68

Remark 2.6.3 Notice that when you converted 68 degrees Fahrenheit, you got a value of 20 degrees Celsius. Alternatively, when you converted 20 degrees Celsius, you got 68 degrees Fahrenheit. This indicates that the equation you were given for C and the equation you found for F are inverses.

Finding the Inverse Function (FN6)

Definition 2.6.4 Let f be a function. If there exists a function g such that

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

for all x, then we say f has an **inverse function**, or that g is the **inverse of** f. When a given function f has an inverse function, we usually denote it as f^{-1}, which is read as "f inverse". $\Diamond$

Finding the Inverse Function (FN6)

Remark 2.6.5 An inverse is a function that "undoes" another function. For any input in the domain, the function g will reverse the process of f.

Activity 2.6.6 It is important to note that in Definition 2.6.4 we say "if there exists a function," but we don't guarantee that this is always the case. How can we determine whether a function has a corresponding inverse or not? Consider the following two functions f and g represented by the tables.

Table 2.6.7

x	$f(x)$
0	6
1	4
2	3
3	4
4	6

Table 2.6.8

x	$g(x)$
0	3
1	1
2	4
3	2
4	0

(a) Use the definition of $g(x)$ in Table 2.6.8 to find an x such that $g(x) = 4$.

A. $x = 0$

B. $x = 1$

C. $x = 2$

D. $x = 3$

E. $x = 4$

(b) Is it possible to reverse the input and output rows of the function $g(x)$ and have the new table result in a function?

(c) Use the definition of $f(x)$ in Table 2.6.7 to find an x such that $f(x) = 4$.

A. $x = 0$

B. $x = 1$

C. $x = 2$

D. $x = 3$

E. $x = 4$

(d) Is it possible to reverse the input and output rows of the function $f(x)$ and have the new table result in a function?

Remark 2.6.9 Some functions, like $f(x)$ in Table 2.6.7, have a given output value that corresponds to two or more input values: $f(0) = 6$ and $f(4) = 6$. If we attempt to reverse the process of this function, we have a situation where the new input 6 would correspond to two potential outputs.

Finding the Inverse Function (FN6)

Definition 2.6.10 A **one-to-one function** is a function in which each output value corresponds to exactly one input. ◊

Finding the Inverse Function (FN6)

Remark 2.6.11 A function must be one-to-one in order to have an inverse.

Activity 2.6.12 Consider the function $f(x) = \dfrac{x-5}{3}$.

(a) When you evaluate this expression for a given input value of x, what operations do you perform and in what order?

A. divide by 3, subtract 5

B. subtract 5, divide by 3

C. add 5, multiply by 3

D. multiply by 3, add 5

(b) When you "undo" this expression to solve for a given ouput value of y, what operations do you perform and in what order?

A. divide by 3, subtract 5

B. subtract 5, divide by 3

C. add 5, multiply by 3

D. multiply by 3, add 5

(c) This set of operations reverses the process for the original function, so can be considered the inverse function. Write an equation to express the inverse function f^{-1}.

A. $f^{-1}(x) = \dfrac{x}{3} - 5$

B. $f^{-1}(x) = \dfrac{x-5}{3}$

C. $f^{-1}(x) = 5(x+3)$

D. $f^{-1}(x) = 3x+5$

(d) Check your answer to the previous question by finding $f(f^{-1}(x))$ and $f^{-1}(f(x))$.

Observation 2.6.13 To find the inverse of a one-to-one function, perform the reverse operations in the opposite order.

Activity 2.6.14 Let's look at an alternate method for finding an inverse by solving the function for x and then interchanging the x and y.

$$h(x) = \frac{x}{x+1}$$

(a) Interchange the variables x and y.

A. $y = \dfrac{x}{x+1}$

B. $x = \dfrac{y}{x+1}$

C. $x = \dfrac{y}{y+1}$

D. $x = \dfrac{x}{y+1}$

(b) Eliminate the denominator.

A. $y(x+1) = x$

B. $x(x+1) = y$

C. $x(y+1) = x$

D. $x(y+1) = y$

(c) Distribute and gather the y terms together.

A. $yx + y = x$

B. $x^2 + x = y$

C. $xy - y = -x$

D. $xy = 0$

(d) Write the inverse function, by factoring and solving for y.

A. $h^{-1}(x) = \dfrac{x}{x-1}$

B. $h^{-1}(x) = \dfrac{x}{1-x}$

C. $h^{-1}(x) = \dfrac{-x}{1-x}$

D. $h^{-1}(x) = \dfrac{x+1}{x}$

Activity 2.6.15 Find the inverse of each function, using either method. Check your answer using function composition.

(a) $g(x) = \dfrac{4x-1}{7}$

A. $g^{-1}(x) = \dfrac{7x+1}{4}$

B. $g^{-1}(x) = \dfrac{7x}{4} + 1$

C. $g^{-1}(x) = \dfrac{4x+1}{7}$

D. $g^{-1}(x) = \dfrac{7}{4x-1}$

(b) $f(x) = 3 - \sqrt{x+5}$

A. $f^{-1}(x) = 3 + \sqrt{x-5}$

B. $f^{-1}(x) = (x-3)^2 + 5$

C. $f^{-1}(x) = \dfrac{1}{3-\sqrt{x+5}}$

D. $f^{-1}(x) = (3-x)^2 - 5$

Chapter 3

Linear Functions (LF)

Objectives

How do we model scenarios that have a constant rate of change?
By the end of this chapter, you should be able to...

1. Determine the average rate of change of a given function over a given interval. Find the slope of a line.

2. Determine an equation for a line when given two points on the line and when given the slope and one point on the line. Express these equations in slope-intercept or point-slope form and determine the slope and y-intercept of a line given an equation.

3. Graph a line given its equation or some combination of characteristics, such as points on the graph, a table of values, the slope, or the intercepts.

4. Use slope relationships to determine whether two lines are parallel or perpendicular, and find the equation of lines parallel or perpendicular to a given line through a given point.

5. Build linear models from verbal descriptions, and use the models to establish conclusions, including by contextualizing the meaning of slope and intercept parameters.

6. Solve a system of two linear equations in two variables.

7. Solve questions involving applications of systems of equations.

3.1 Slope and Average Rate of Change (LF1)

Objectives

- Determine the average rate of change of a given function over a given interval. Find the slope of a line.

Remark 3.1.1 This section will explore ideas around average rate of change and slope. To help us get started, let's take a look at a context in which these ideas can be helpful.

Activity 3.1.2 Robert came home one day after school to a very hot house! When he got home, the temperature on the thermostat indicated that it was 85 degrees! Robert decided that was too hot for him, so he turned on the air conditioner. The table of values below indicate the temperature of his house after turning on the air conditioner.

Table 3.1.3

Time (minutes)	Temperature (degrees Fahrenheit)
0	85
1	84.3
2	83.6
3	82.9
4	82.2
5	81.5
6	80.8

(a) How much did the temperature change from 0 to 2 minutes?

A. The temperature decreased by 0.7 degrees

B. The temperature decreased by 1.4 degrees

C. The temperature increased by 0.7 degrees

D. The temperature increased by 1.4 degrees>

(b) How much did the temperature change from 4 to 6 minutes?

A. The temperature decreased by 0.7 degrees

B. The temperature decreased by 1.4 degrees

C. The temperature increased by 0.7 degrees

D. The temperature increased by 1.4 degrees>

(c) If Robert wanted to know how much the temperature was decreasing each minute, how could he figure that out?

(d) How would you describe the overall behavior of the temperature of Robert's house?

Remark 3.1.4 Notice in Activity 3.1.2 that the temperature appears to be decreasing at a constant rate (i.e., the temperature decreased 1.4 degrees for every 2-minute interval). Upon further investigation, you might have also noticed that the temperature decreased by 0.7 degrees every minute.

Activity 3.1.5 Refer back to the data Robert collected of the temperature of his house after turning on the air conditioner (Table 3.1.3).

(a) If this pattern continues, what will the temperature be after 8 minutes?

A. 80.1

B. 78.7

C. 80.8

D. 79.4

(b) If this pattern continues, how long will it take for Robert's house to reach 78 degrees?

A. 12 minutes

B. 9 minutes

C. 10 minutes

D. 11 minutes

Remark 3.1.6 An average rate of change helps us to see and understand how a function is generally behaving. For example, in Activity 3.1.2 and Activity 3.1.5, we began to see how the temperature of Robert's house was decreasing every minute the air conditioner was on. In other words, when looking at average rate of change, we are comparing how one quantity is changing with respect to something else changing.

Definition 3.1.7 An **average rate of change** of a function calculates the amount of change in one item divided by the corresponding amount of change in another.

To calculate the average rate of change for any function $f(x)$, we pick two points, a and b, and evaluate the function at those two points. We then find the difference between the y-values and x-values to calcuate the average rate of change:

Recall that we can use function notation to describe x- and y-values. $f(b)$, for instance represents the y-value when plugging in a value for x (or b).

$$\frac{f(b) - f(a)}{b - a}.$$

$\diamondsuit$

Activity 3.1.8 Use the table below to answer the questions.

Table 3.1.9

x	$f(x)$
-5	28
-4	19
-3	12
-2	7
-1	4

(a) Applying Definition 3.1.7, what is the average rate of change when $x = -5$ to $x = -2$?

A. $\frac{1}{7}$

B. -3

C. -7

D. 7

(b) What is the average rate of change on the interval $[-4, -1]$?

A. -5

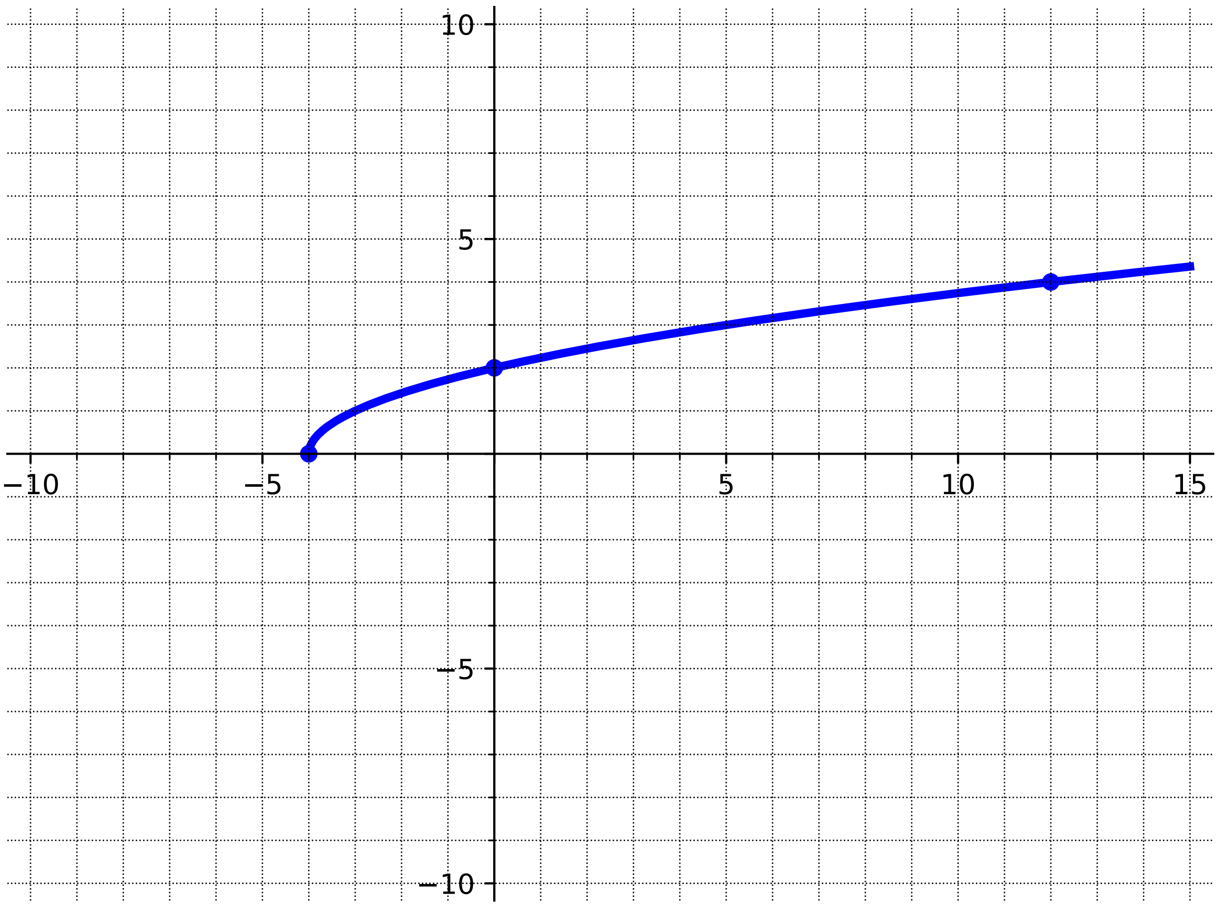
B. -3

C. 5

D. 3

(c) Does this function have a constant average rate of change?

Activity 3.1.10 Use the graph to calculate the average rate of change on the given intervals.



(a) Applying Definition 3.1.7, what is the average rate of change on the interval $[-4, 0]$?

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. 2

(b) What is the average rate of change on the interval $[0, 12]$?

A. $\frac{1}{6}$

B. -6

C. $-\frac{1}{6}$

D. 6

Activity 3.1.11 Just like with tables and graphs, you should be able to find the average rate of change when given a function. For this activity, use the function

$$f(x) = -3x^2 - 1$$

to answer the following questions.

(a) Applying Definition 3.1.7, what is the average rate of change on the interval $[-2, 3]$?

A. $\frac{41}{5}$

B. -3

C. $-\frac{1}{3}$

D. $\frac{5}{41}$

(b) What is the average rate of change on the interval $[0, 4]$?

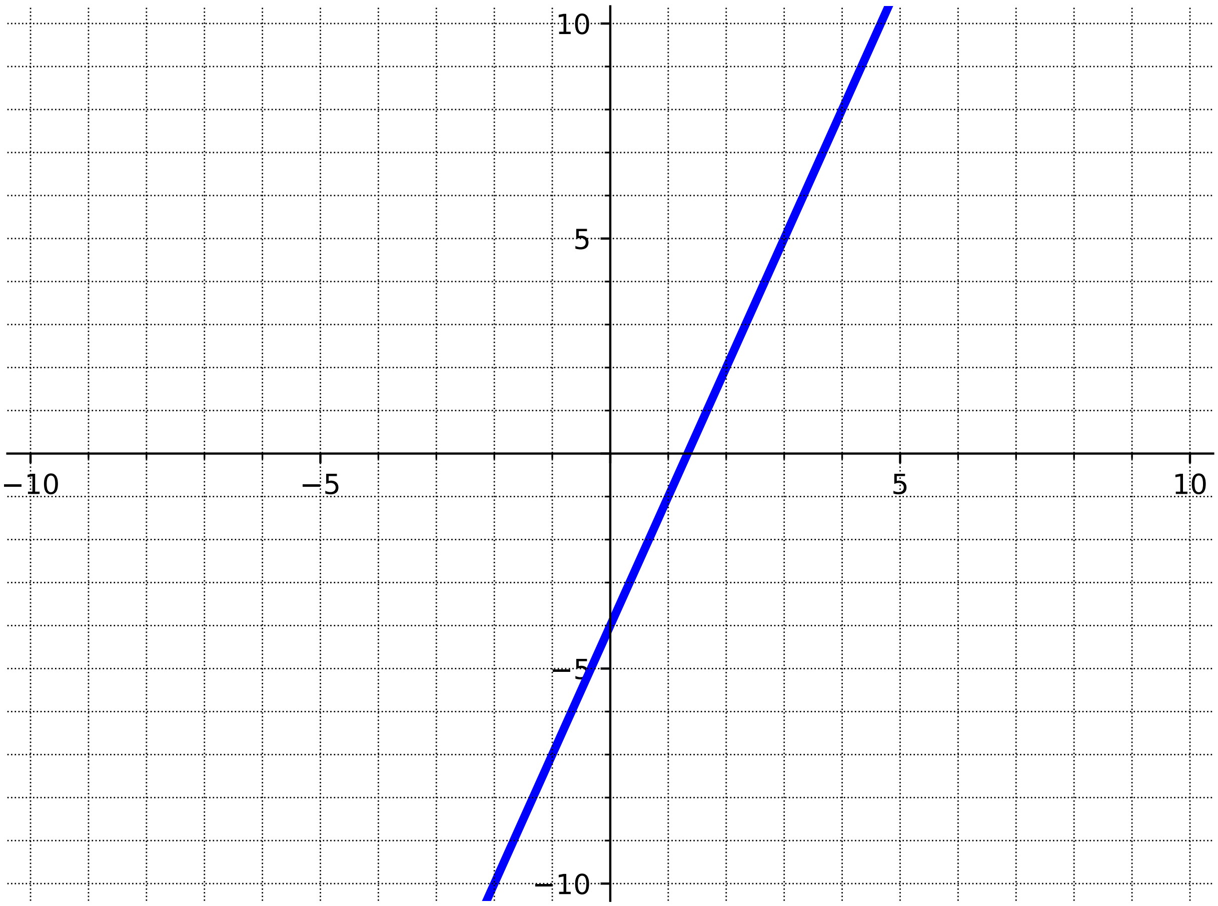
A. $\frac{2}{25}$

B. -50

C. $-\frac{1}{12}$

D. -12

Activity 3.1.12 Use the given graph of the function, $f(x) = 3x - 4$, to investigate the average rate of change of a linear function.



(a) What is the average rate of change on the interval $[-2, 0]$?

A. $\frac{1}{3}$

B. 3

C. $-\frac{1}{7}$

D. -7

(b) What is the average rate of change on the interval $[-1, 5]$? Notice that you cannot see the point at $x = 5$. How could you use the equation of the line to determine the y-value when $x = 5$?

A. 3

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. -3

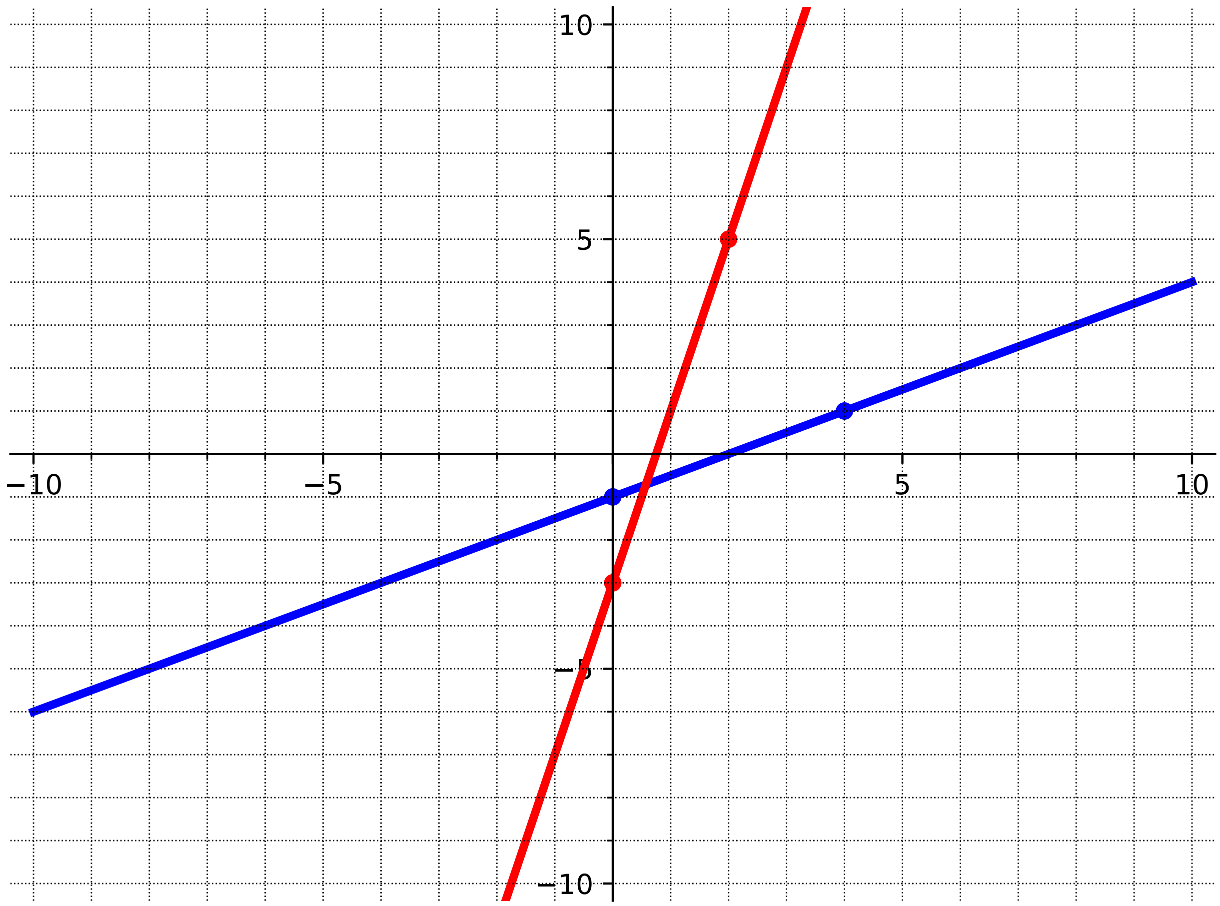
(c) Based on your observations in parts a and b, what do you think will be the average rate of change on the interval $[5, 25]$?

Remark 3.1.13 Notice in Activity 3.1.12, the average rate of change was the same regardless of which interval you were given. But in Activity 3.1.10, the average rate of change was not the same across different intervals.

Slope and Average Rate of Change (LF1)

Definition 3.1.14 The **slope** of a line has a constant that represents the direction and steepness of the line. For a linear function, the slope never changes - meaning it has a constant average rate of change. $\Diamond$

Activity 3.1.15 The steepness of a line depends on the vertical and horizontal distances between two points on the line. Use the graph below to compare the steepness, or slope, of the two lines.



(a) What is the vertical distance between the two points on the red line?

A. 2

B. 4

C. 8

D. $\frac{1}{2}$

(b) What is the horizontal distance between the two points on the red line?

A. 2

B. 4

C. 8

D. $\frac{1}{2}$

(c) Using information from parts a and b, what value could we use to describe the steepness of the red line?

A. 2

B. 4

C. 8

D. $\frac{1}{2}$

(d) What is the vertical distance between the two points on the blue line?

A. 2

B. 4

C. 8

D. $\frac{1}{2}$

(e) What is the horizontal distance between the two points on the blue line?

A. 2

B. 4

C. 8

D. $\frac{1}{2}$

(f) Using information from parts d and e, what value could we use to describe the steepness of the blue line?

A. 2

B. 4

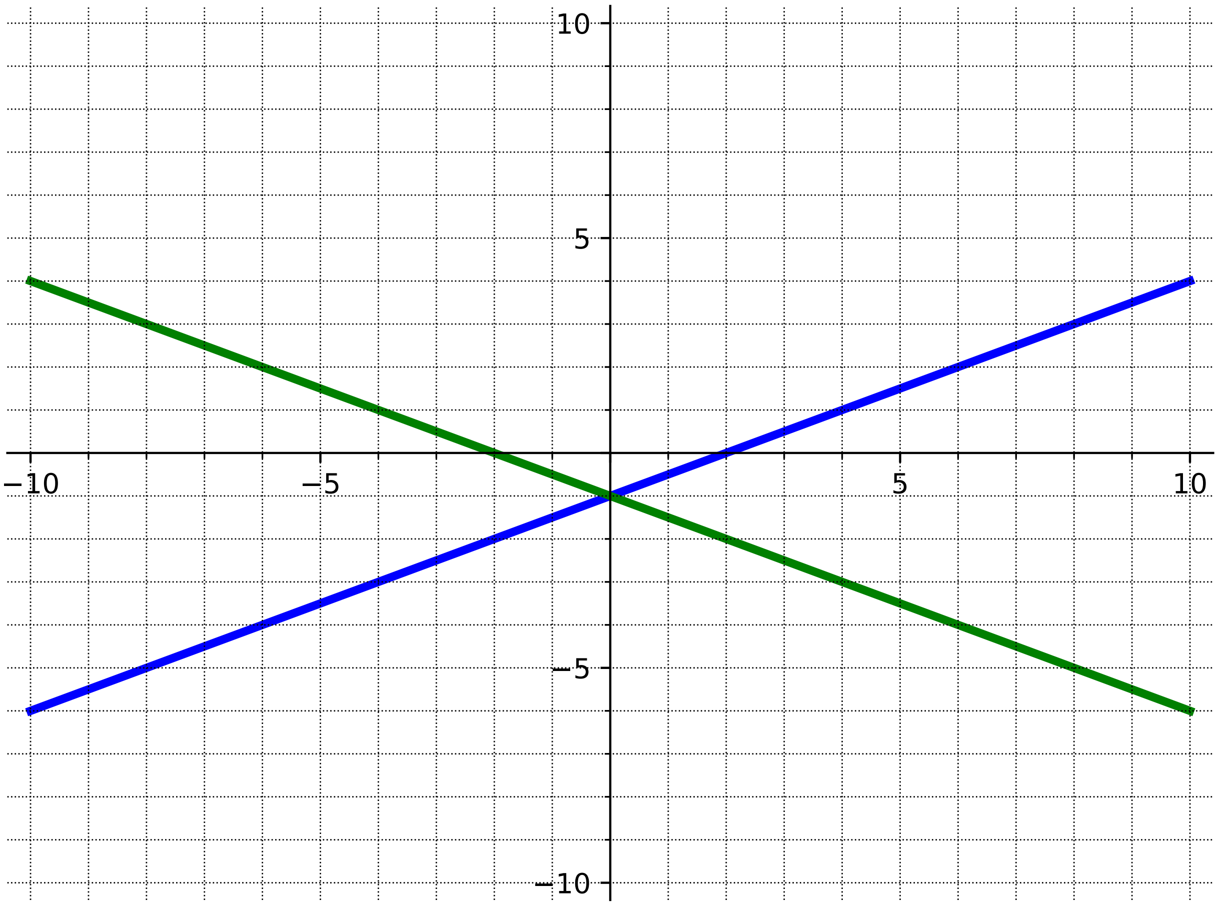
C. 8

D. $\frac{1}{2}$

(g) Which line is the steepest?

Remark 3.1.16 The steepness, or slope, of a line can be found by the change in y (the vertical distance between two points on the line) divided by the change in x (the horizontal distance between two points on the line). Slope can be calculated as "rise over run." Slope is a way to describe the steepness of a line. The red line in Activity 3.1.15 has a larger value for it's slope than the blue line. Thus, the red line is steeper than the blue line.

Activity 3.1.17 Now that we know how to find the slope (or steepness) of a line, let's look at other properties of slope. Use the graph below to answer the following questions.



(a) What is the slope of the blue line?

A. $\frac{1}{2}$

B. 2

C. $-\frac{1}{2}$

D. -2

(b) What is the slope of the green line?

A. $\frac{1}{2}$

B. 2

C. $-\frac{1}{2}$

D. -2

(c) How are the slopes of the lines similar?

(d) How are the slopes of the lines different?

Remark 3.1.18 Notice in Activity 3.1.17 that the slope does not just indicate how steep a line is, but also it's direction. A negative slope indicates that the line is decreasing (from left to right) and a positive slope indicates that the line is increasing (from left to right).

Activity 3.1.19 Suppose $(-3, 7)$ and $(7, 2)$ are two points on a line.

(a) Plot these points on a graph and find the slope by using "rise over run."

A. $\frac{1}{2}$

B. 2

C. $-\frac{1}{2}$

D. -2

(b) Now calculate the slope by using the change in y over the change in x.

A. $\frac{1}{2}$

B. 2

C. $-\frac{1}{2}$

D. -2

(c) What do you notice about the slopes you got in parts a and b?

Remark 3.1.20 We can calculate slope (m) by finding the change in y and dividing by the change in x. Mathematically, this means that when given (x_1, y_1) and (x_2, y_2),

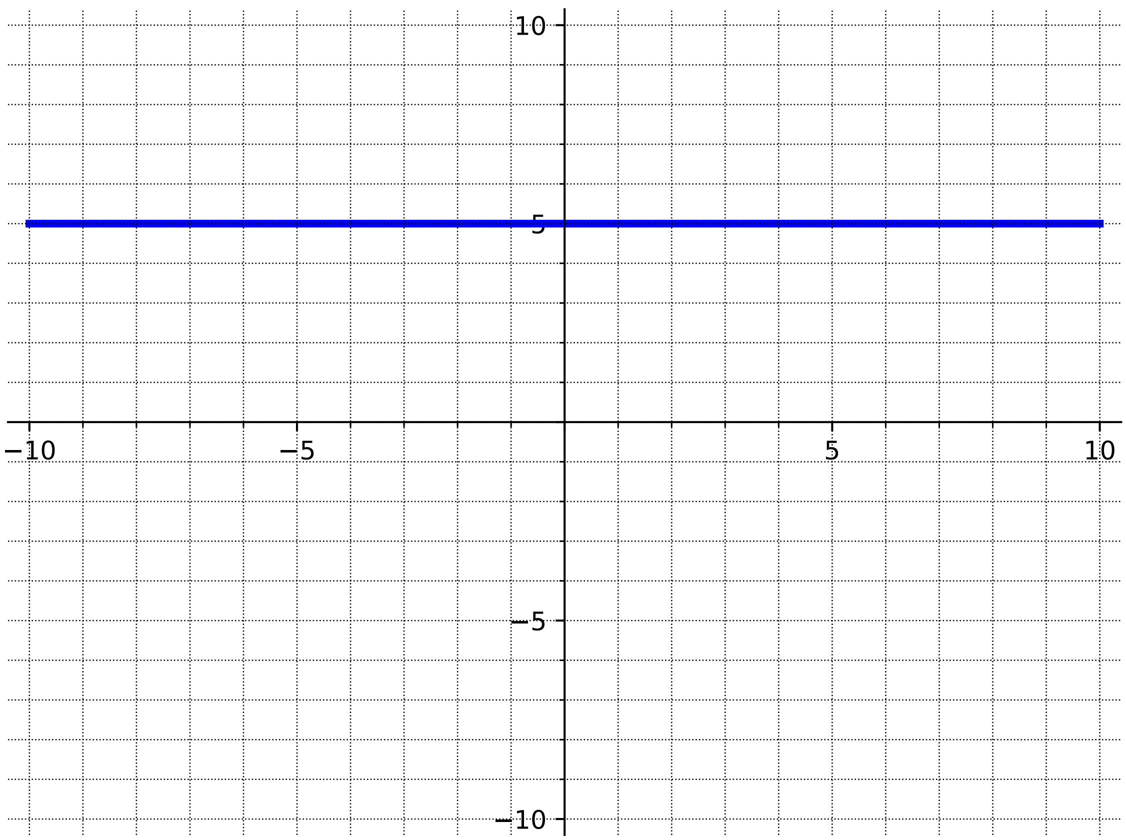
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Activity 3.1.21 Calculate the slope of each representation of a line using the slope formula.

(a)

x	$f(x)$
-2	-7
-1	-4
0	-1
1	2
2	5

(b)



(c) $(-4, 7)$ and $(-4, 1)$

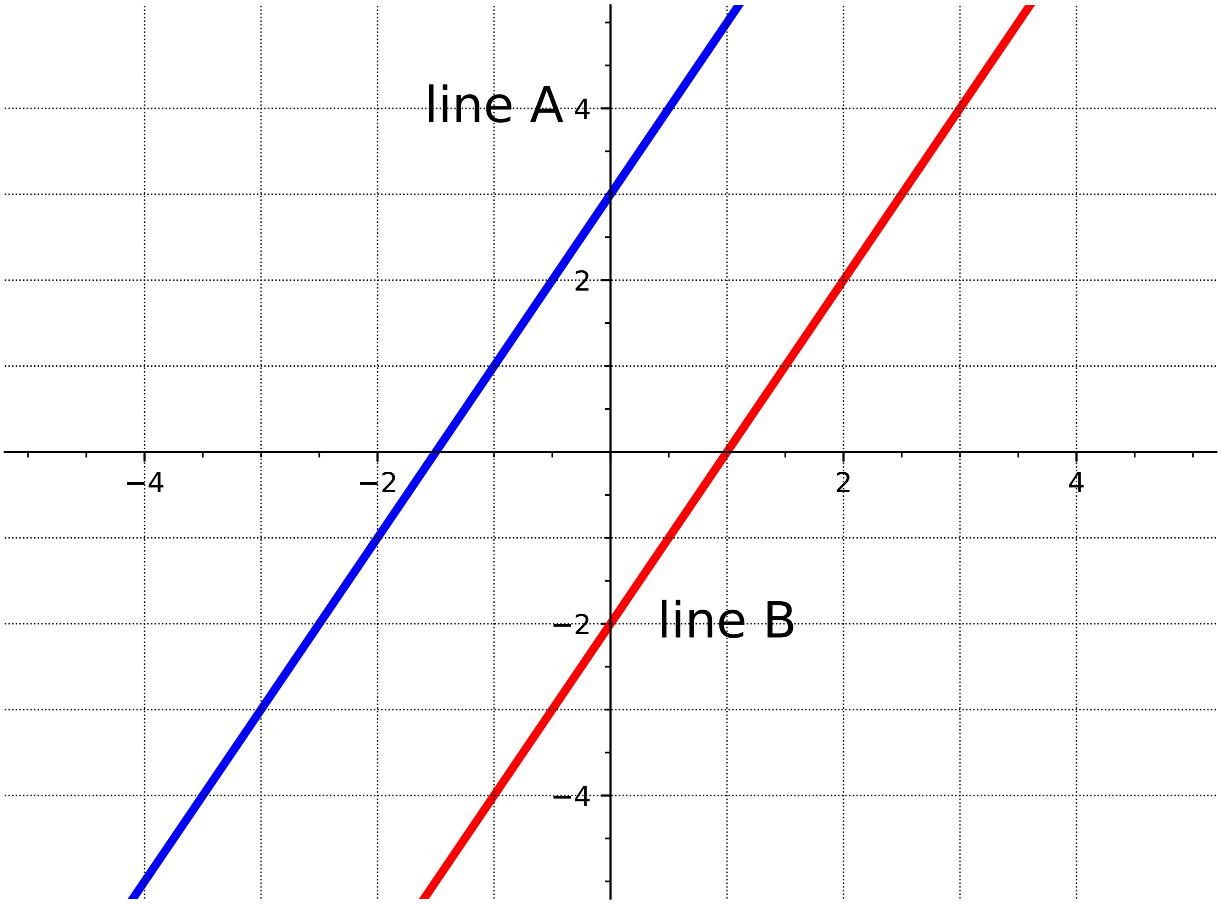
Remark 3.1.22 In Activity 3.1.21, there were slopes that were 0 and undefined. When a line is vertical, the slope is undefined. This means that there is only a vertical distance between two points and there is no horizontal distance. When a line is horizontal, the slope is 0. This means that the line never rises vertically, giving a vertical distance of zero.

3.2 Equations of Lines (LF2)

Objectives

- Determine an equation for a line when given two points on the line and when given the slope and one point on the line. Express these equations in slope-intercept or point-slope form and determine the slope and y-intercept of a line given an equation.

Activity 3.2.1 Consider the graph of two lines.



(a) Find the slope of line A.

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

(b) Find the slope of line B.

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

(c) Find the y-intercept of line A.

A. -2

B. -1.5

C. 1

D. 3

(d) Find the y-intercept of line B.

A. -2

B. -1.5

C. 1

D. 3

(e) What is the same about the two lines?

(f) What is different about the two lines?

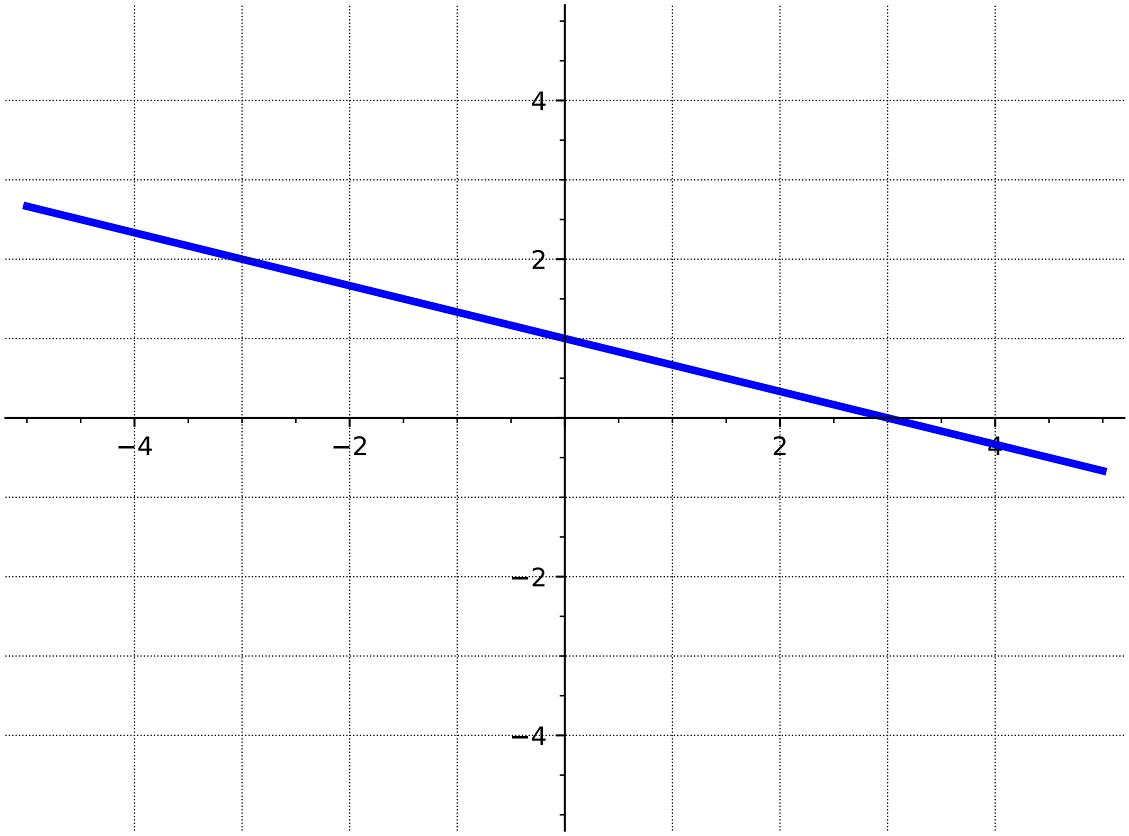
Remark 3.2.2 Notice that in Activity 3.2.1 the lines have the same slope but different y-intercepts. It is not enough to just know one piece of information to determine a line, you need both a slope and a point.

Definition 3.2.3 Linear functions can be written in **slope-intercept form**

$$f(x) = mx + b$$

where b is the y-intercept (or starting value) and m is the slope (or constant rate of change). $\Diamond$

Activity 3.2.4 Write the equation of each line in slope-intercept form.



(a)

A. $y = -3x + 1$

B. $y = -x + 3$

C. $y = -\dfrac{1}{3}x + 1$

D. $y = -\dfrac{1}{3}x + 3$

(b) The slope is 4 and the y-intercept is $(0, -3)$.

A. $f(x) = 4x - 3$

B. $f(x) = 3x - 4$

C. $f(x) = -4x + 3$

D. $f(x) = 4x + 3$

(c) Two points on the line are $(0, 1)$ and $(2, 4)$.

A. $y = 2x + 1$

B. $y = -\dfrac{3}{2}x + 4$

C. $y = \dfrac{3}{2}x + 1$

D. $y = \dfrac{3}{2}x + 4$

(d)

x	$f(x)$
-2	-8
0	-2
1	1
4	10

A. $f(x) = -3x - 2$

B. $f(x) = -\dfrac{1}{3}x - 2$

C. $f(x) = 3x + 1$

D. $f(x) = 3x - 2$

Activity 3.2.5 Let's try to write the equation of a line given two points that don't include the y-intercept.

(a) Plot the points $(2, 1)$ and $(-3, 4)$.

(b) Find the slope of the line joining the points.

A. $-\frac{5}{3}$

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. -3

(c) When you draw a line connecting the two points, it's often hard to draw an accurate enough graph to determine the y-intercept of the line exactly. Let's use the slope-intercept form and one of the given points to solve for the y-intercept. Try using the slope and one of the points on the line to solve the equation $y = mx + b$ for b.

A. 2

B. $\frac{11}{5}$

C. $\frac{5}{2}$

D. 3

(d) Write the equation of the line in slope-intercept form.

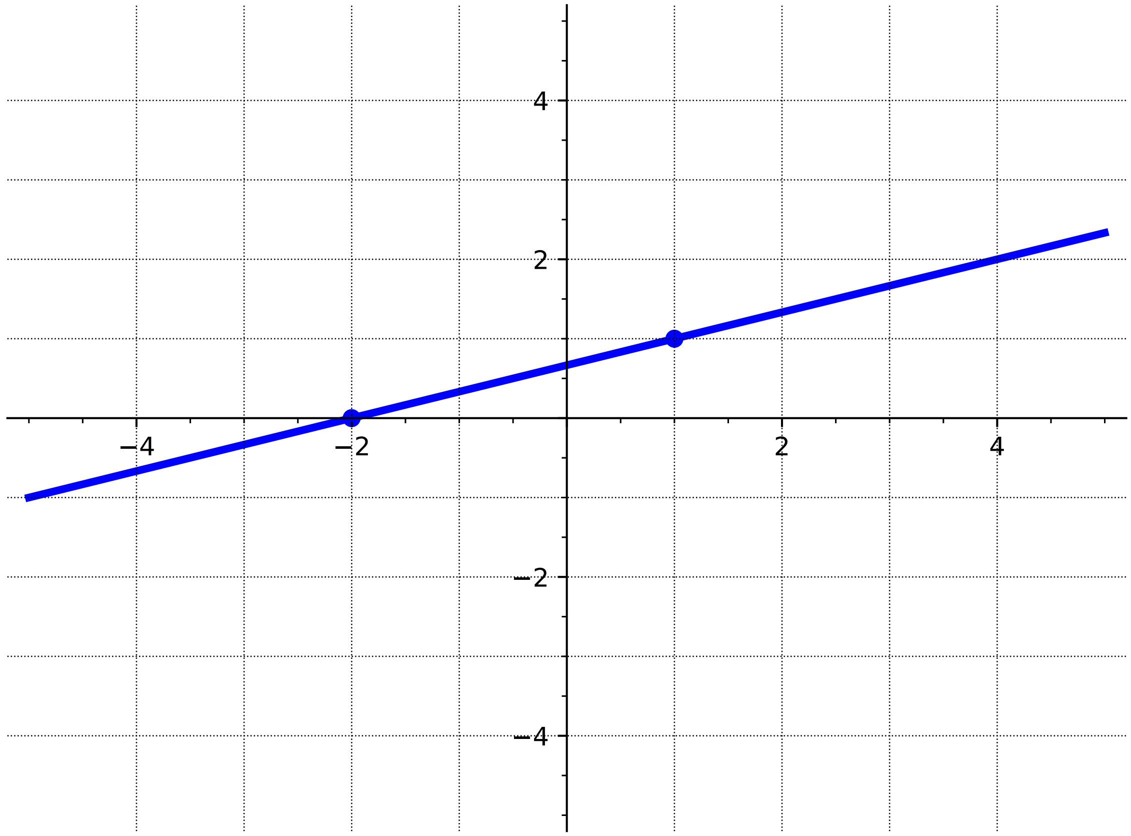
Remark 3.2.6 It would be nice if there was another form of the equation of a line that works for any points and does not require the y-intercept.

Definition 3.2.7 Linear functions can be written in **point-slope form**

$$y - y_0 = m(x - x_0)$$

where (x_0, y_0) is any point on the line and m is the slope. $\diamondsuit$

Activity 3.2.8 Write an equation of each line in point-slope form.



(a)

A. $y = \frac{1}{3}x + \frac{2}{3}$

B. $y - 1 = 3(x - 1)$

C. $y - 1 = \frac{1}{3}(x - 1)$

D. $y + 2 = \frac{1}{3}(x + 2)$

E. $y = \frac{1}{3}(x + 2)$

(b) The slope is 4 and $(-1, -7)$ is a point on the line.

A. $y + 7 = 4(x + 1)$

B. $y - 7 = 4(x - 1)$

C. $y + 1 = 4(x + 7)$

D. $y - 4 = 7(x - 1)$

(c) Two points on the line are $(1, 0)$ and $(2, -4)$.

A. $y = -4x + 1$

B. $y - 0 = -2(x - 1)$

C. $y + 4 = -4(x - 2)$

D. $y + 4 = -3(x - 2)$

(d)

x	$f(x)$
-2	-8
1	1
4	10

A. $y + 8 = 3(x - 2)$

B. $y - 1 = -\frac{1}{3}(x - 1)$

C. $y + 8 = -\frac{1}{3}(x + 2)$

D. $y - 10 = 3(x - 4)$

Activity 3.2.9 Consider again the two points from Activity 3.2.5, $(2, 1)$ and $(-3, 4)$.

(a) Use point-slope form to find an equation of the line.

A. $y = -\dfrac{3}{5}x + \dfrac{11}{5}$

B. $y - 1 = -\dfrac{3}{5}(x - 2)$

C. $y - 4 = -\dfrac{3}{5}(x + 3)$

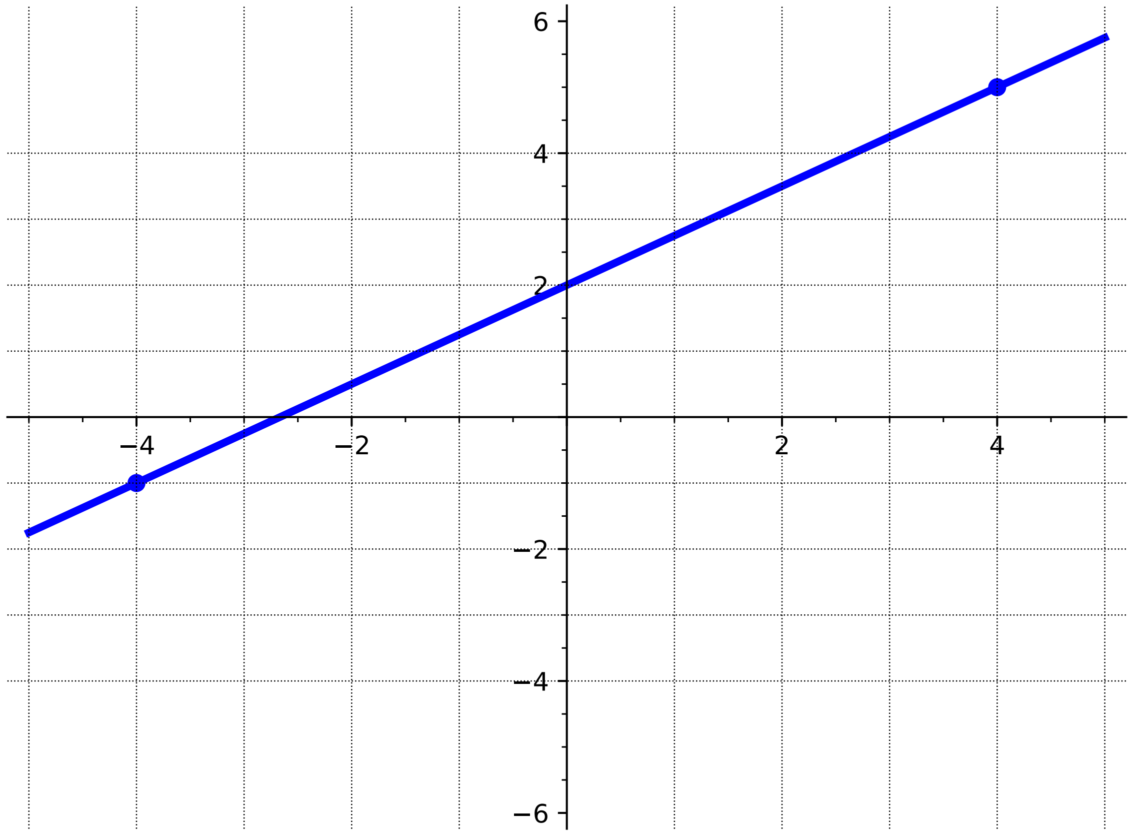
D. $y - 2 = -\dfrac{3}{5}(x - 1)$

(b) Solve the point-slope form of the equation for y to rewrite the equation in slope-intercept form. Identify the slope and intercept of the line.

Remark 3.2.10 Notice that it was possible to use either point to find an equation of the line in point-slope form. But, when rewritten in slope-intercept form the equation is unique.

Activity 3.2.11 For each of the following lines, determine which form (point-slope or slope-intercept) would be "easier" and why. Then, write the equation of each line.

(a)



(b) The slope is $-\frac{1}{2}$ and $(1, -3)$ is a point on the line.

(c) Two points on the line are $(0, 3)$ and $(2, 0)$.

Remark 3.2.12 It is always possible to use both forms to write the equation of a line and they are both valid. Although, sometimes the given information lends itself to make one form easier.

Activity 3.2.13 Write the equation of each line.

(a) The slope is 0 and $(-1, -7)$ is a point on the line.

A. $y = -7$

B. $y = 7x$

C. $y = -x$

D. $x = -1$

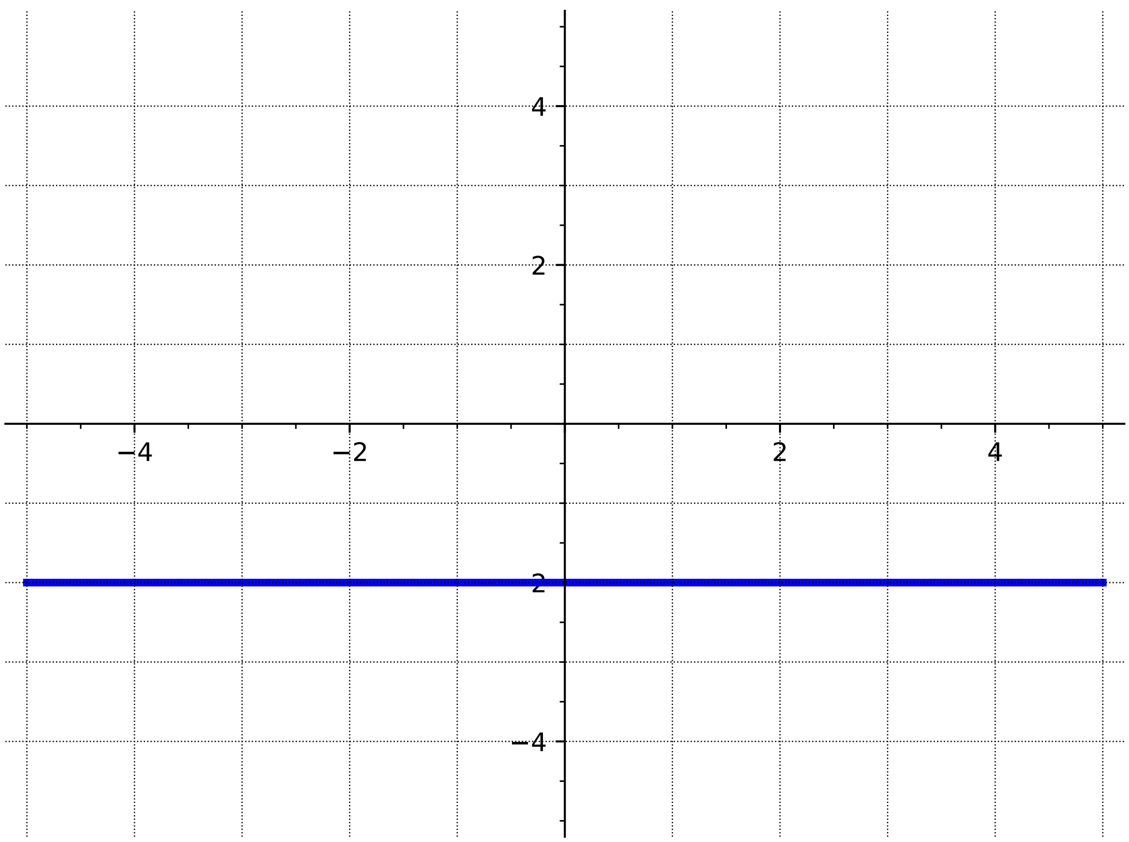
(b) Two points on the line are $(3, 0)$ and $(3, 5)$.

A. $y = 3x + 3$

B. $y = 3x + 5$

C. $x = 3$

D. $y = 3$



(c) A. $x = -2$

B. $y - 2 = x$

C. $y = -2x - 2$

D. $y = -2$

Equations of Lines (LF2)

Definition 3.2.14 A **horizontal line** has a slope of zero and has the form $y = k$ where k is a constant. A **vertical line** has an undefined slope and has the form $x = h$ where h is a constant. $\diamondsuit$

Definition 3.2.15 The equation of a line can also be written in **standard form**. Standard form looks like $Ax + By = C$. $\Diamond$

Remark 3.2.16 It is possible to rearrange a line written in standard form to slope-intercept form, by solving for y.

Activity 3.2.17 Given a line in standard form

$$5x + 4y = 2.$$

Find the slope and y-intercept of the line.

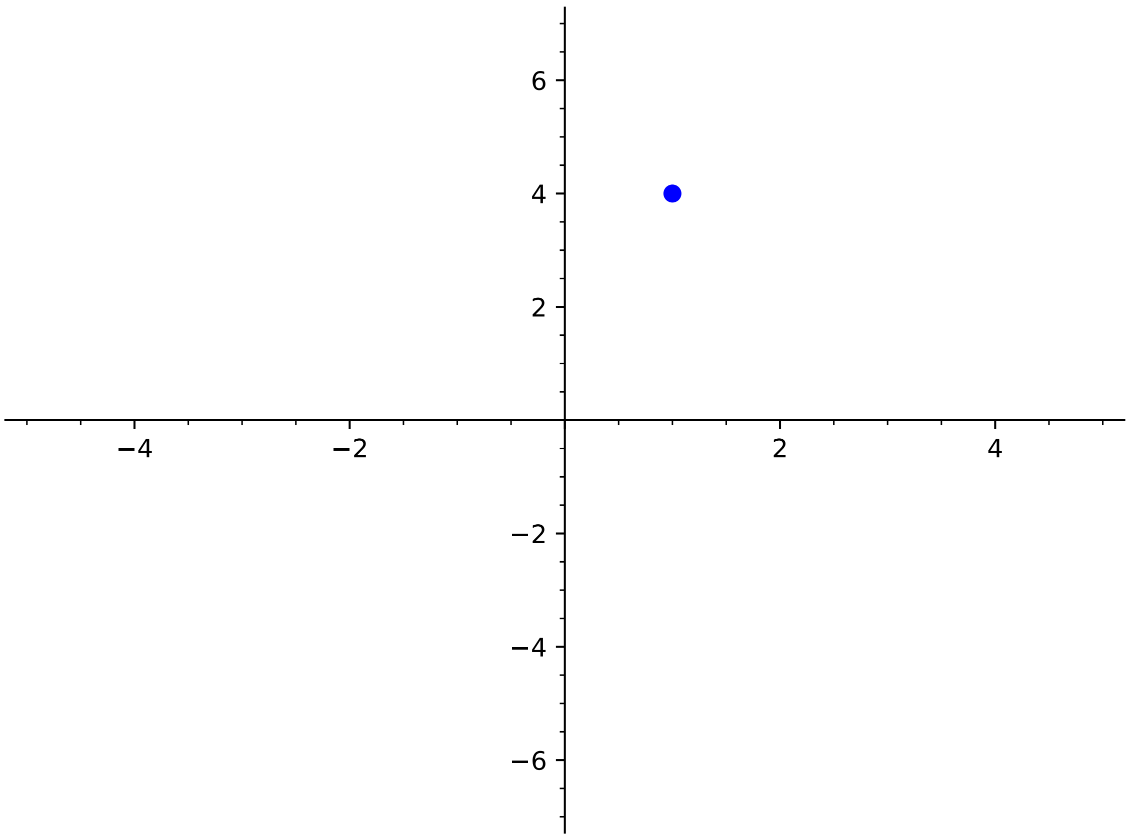
3.3 Graphs of Linear Equations (LF3)

Objectives

- Graph a line given its equation or some combination of characteristics, such as points on the graph, a table of values, the slope, or the intercepts.

Activity 3.3.1

(a) Draw a line that goes through the point $(1, 4)$.



(b) Was this the only possible line that goes through the point $(1, 4)$?

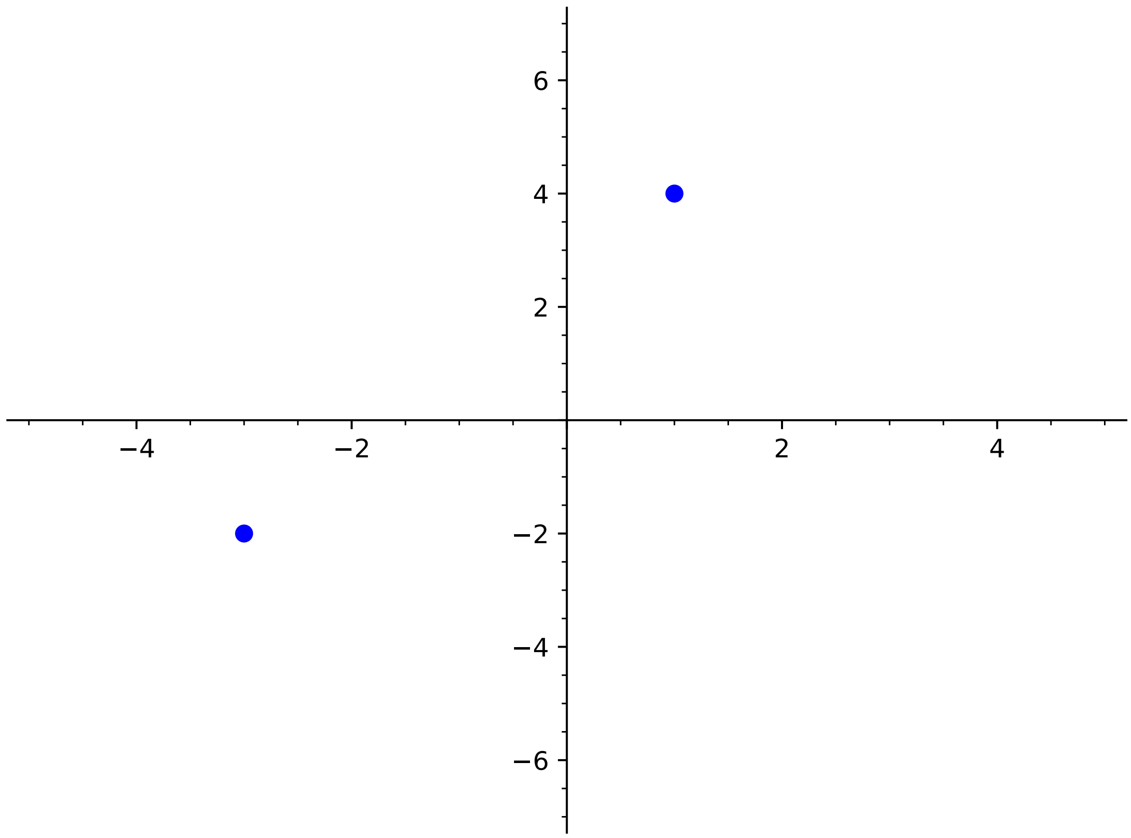
A. Yes. The line is unique.

B. No. There is exactly one more line possible.

C. No. There are a lot of lines that go through $(1, 4)$.

D. No. There are an infinite number of lines that go through $(1, 4)$.

(c) Now draw a line that goes through the points $(1, 4)$ and $(-3, -2)$.



(d) Was this the only possible line that goes through the points $(1, 4)$ and $(-3, -2)$?

A. Yes. The line is unique.

B. No. There is exactly one more line possible.

C. No. There are a lot of lines that go through $(1, 4)$ and $(-3, -2)$.

D. No. There are an infinite number of lines that go through $(1, 4)$ and $(-3, -2)$.

Observation 3.3.2 If you are given two points, then you can always graph the line containing them by plotting them and connecting them with a line.

Activity 3.3.3

(a) Graph the line containing the points $(-7, 1)$ and $(6, -2)$.

(b) Graph the line containing the points $(-3, 0)$ and $(0, 8)$.

(c) Graph the line given by the table below.

x	y
-3	-12
-2	-9
-1	-6
0	-3
1	0
2	3

(d) Let's say you are given a table that listed six points that are on the same line. How many of those points are necessary to use to graph the line?

A. One point is enough.

B. Two points are enough.

C. Three points are enough.

D. You need to plot all six points.

E. You can use however many you want.

Remark 3.3.4 In Activity 3.3.3, we were given at least two points in each question. However, sometimes we are not directly given two points to graph a line. Instead we are given some combination of characteristics about the line that will help us *find* two points. These characteristics could include a point, the intercepts, the slope, or an equation.

Activity 3.3.5 A line has a slope of $-\frac{1}{3}$ and its y-intercept is 4.

(a) We were given the y-intercept. What point does that correspond to?

A. $(4, 0)$

B. $(0, 4)$

C. $\left(4, -\dfrac{1}{3}\right)$

D. $\left(-\dfrac{1}{3}, 4\right)$

(b) After we plot the y-intercept, how can we use the slope to find another point?

A. Start at the y-intercept, then move up one space and to the left three spaces to find another point.

B. Start at the y-intercept, then move up one space and to the right three spaces to find another point.

C. Start at the y-intercept, then move down one space and to the left three spaces to find another point.

D. Start at the y-intercept, then move down one space and to the right three spaces to find another point.

(c) Graph the line that has a slope of $-\frac{1}{3}$ and its y-intercept is 4.

Activity 3.3.6 A line is given by the equation $y = -2x + 5$.

(a) What form is the equation given in?

A. Standard form

B. Point-slope form

C. Slope-intercept form

D. The form it is in doesn't have a name.

(b) The form gives us one point right away: the y-intercept. Which of the following is the y-intercept?

A. $(-2, 0)$

B. $(0, -2)$

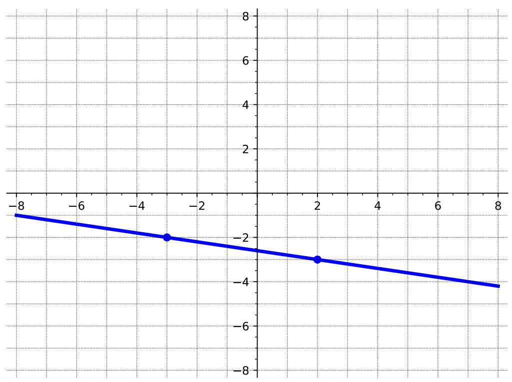
C. $(5, 0)$

D. $(0, 5)$

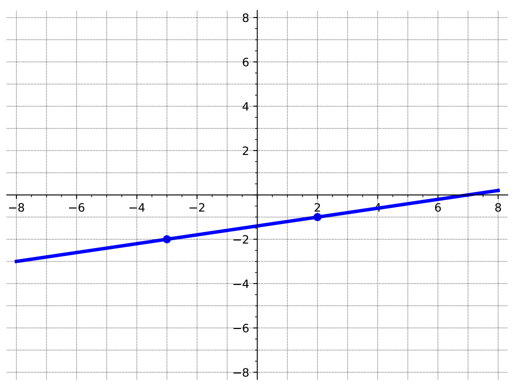
(c) After we plot the y-intercept, we can use the slope to find another point. Find another point and graph the resulting line.

Activity 3.3.7 A line contains the point $(-3,-2)$ and has slope $\frac{1}{5}$. Which of the following is the graph of that line?

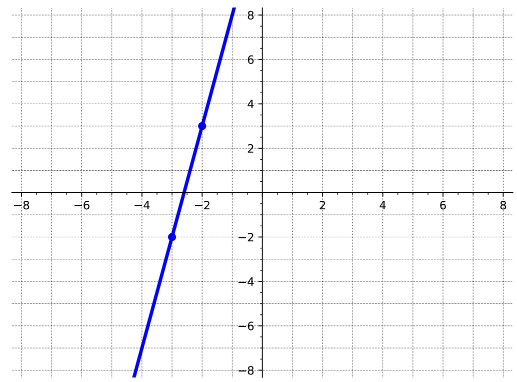
A.



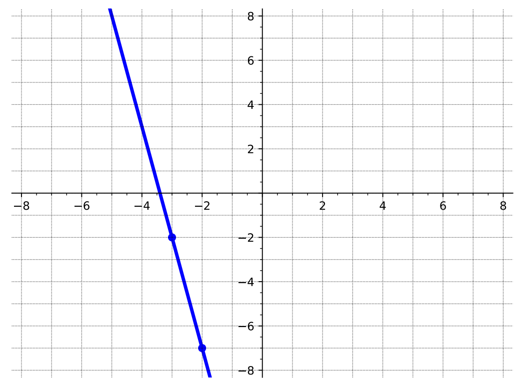
B.



C.



D.



Activity 3.3.8 A line is given by the equation $y - 6 = -4(x + 2)$.

(a) What form is the equation given in?

A. Standard form

B. Point-slope form

C. Slope-intercept form

D. The form it is in doesn't have a name.

(b) The form gives us one point right away. Which of the following is a point on the line?

A. $(-2, -6)$

B. $(-2, 6)$

C. $(2, -6)$

D. $(2, 6)$

(c) After we plot this point, we can use the slope to find another point. Find another point and graph the resulting line.

Activity 3.3.9 Recall from Definition 3.2.14 that the equation of a horizontal line has the form $y = k$ where k is a constant and a vertical line has the form $x = h$ where h is a constant.

(a) Which type of line has a slope of zero?

A. Horizontal

B. Vertical

(b) Which type of line has an undefined slope?

A. Horizontal

B. Vertical

(c) Graph the vertical line that goes through the point $(4, -2)$.

(d) What is the equation of the vertical line through the point $(4, -2)$?

A. $x = 4$

B. $y = 4$

C. $x = -2$

D. $y = -2$

(e) Graph the horizontal line that goes through the point $(4, -2)$.

(f) What is the equation of the horizontal line through the point $(4, -2)$?

A. $x = 4$

B. $y = 4$

C. $x = -2$

D. $y = -2$

Activity 3.3.10 Graph each line described below.

(a) The line containing the points $(-3, 4)$ and $(5, -2)$.

(b) The line whose x-intercept is -2 and whose y-intercept is 7.

(c) The line whose slope is $\frac{2}{5}$ that goes through the point $(4, 6)$.

(d) The line whose slope is $-\frac{1}{3}$ and whose y-intercept is -4.

(e) The vertical line through the point $(-2, -7)$.

(f) The horizontal line through the point $(-6, 3)$.

(g) The line with equation $y = -\frac{5}{3}x - 6$.

(h) The line with equation $y - 5 = \frac{7}{2}(x - 2)$.

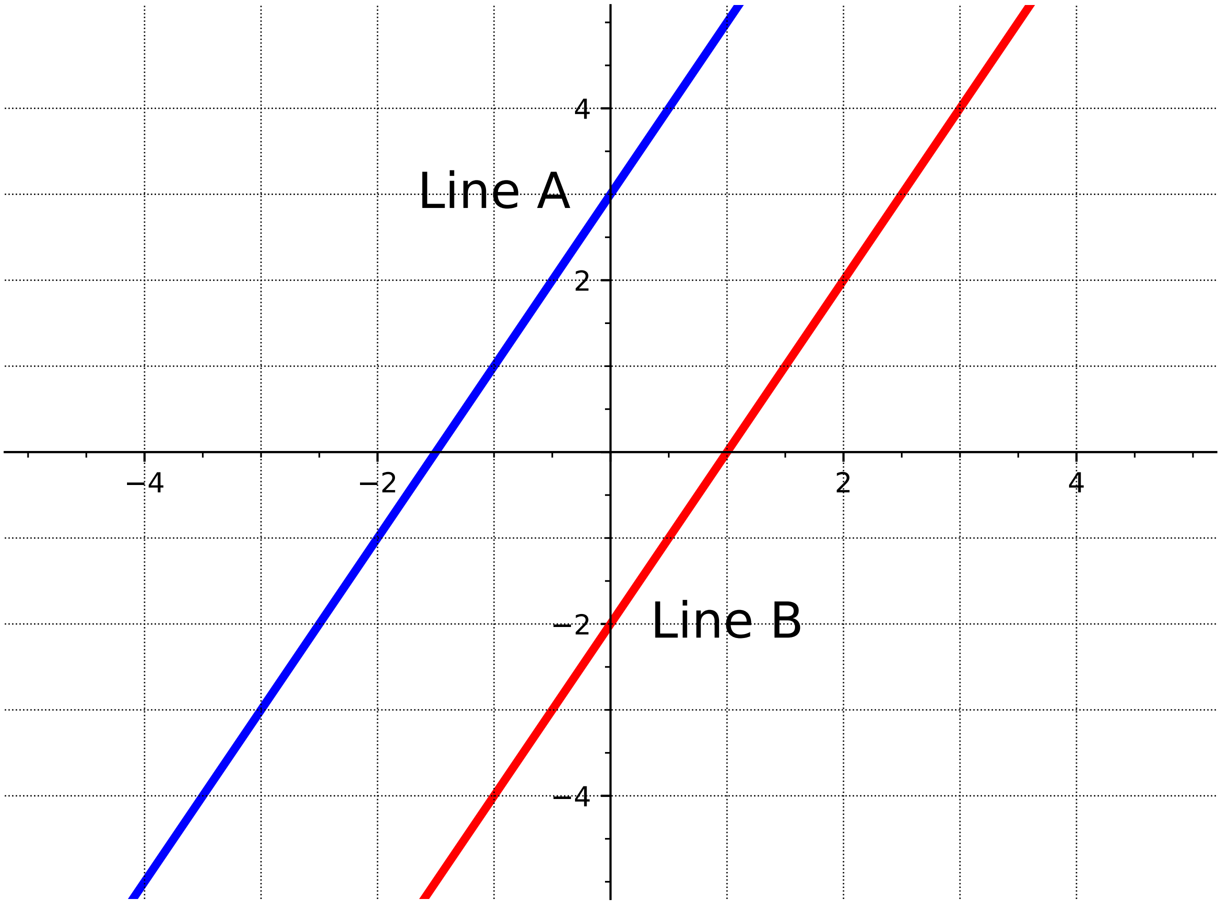
(i) The line with equation $3x - 6y = 8$.

3.4 Parallel and Perpendicular Lines (LF4)

Objectives

- Use slope relationships to determine whether two lines are parallel or perpendicular, and find the equation of lines parallel or perpendicular to a given line through a given point.

Activity 3.4.1 Let's revisit Activity 3.2.1 to investigate special types of lines.



(a) What is the slope of line A?

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

(b) What is the slope of line B?

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

(c) What is the y-intercept of line A?

A. -2

B. -1.5

C. 1

D. 3

(d) What is the y-intercept of line B?

A. -2

B. -1.5

C. 1

D. 3

(e) What is the same about the two lines?

(f) What is different about the two lines?

Remark 3.4.2 Notice that in Activity 3.4.1 the two lines never touch.

Definition 3.4.3 Parallel lines are lines that always have the same distance apart (equidistant) and will never meet. Parallel lines have the same slope, but different y-intercepts. $\Diamond$

Activity 3.4.4 Suppose you have the function,

$$f(x) = -\frac{1}{2}x - 1$$

(a) What is the slope of $f(x)$?

A. -1

B. 2

C. 1

D. $-\frac{1}{2}$

(b) Applying Definition 3.4.3, what would the slope of a line parallel to $f(x)$ be?

A. -1

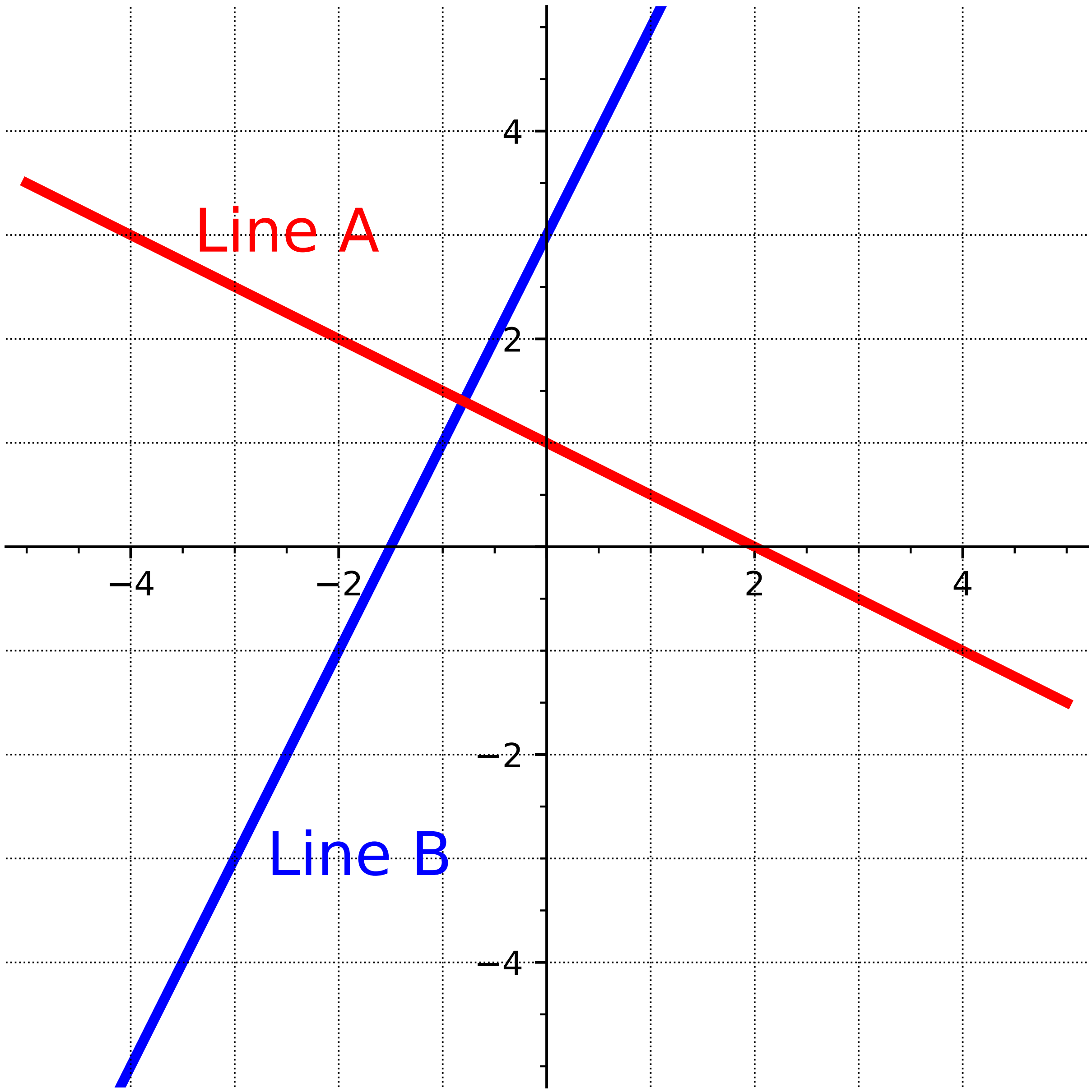
B. 2

C. 1

D. $-\frac{1}{2}$

(c) Find the equation of a line parallel to $f(x)$ that passes through the point $(-4, 2)$.

Activity 3.4.5 Consider the graph of the two lines below.



(a) What is the slope of line A?

A. 3

B. 2

C. $-\frac{1}{2}$

D. -2

(b) What is the slope of line B?

A. 3

B. 2

C. $-\frac{1}{2}$

D. -2

(c) What is the y-intercept of line A?

A. -2

B. $-\frac{1}{2}$

C. 2

D. 3

(d) What is the y-intercept of line B?

A. -2

B. $-\frac{1}{2}$

C. 1

D. 3

(e) If you were to think of slope as ”rise over run,” how would you write the slope of each line?

(f) How would you compare the slopes of the two lines?

Remark 3.4.6 Notice in Activity 3.4.5, that even though the two lines have different slopes, the slopes are somewhat similar. For example, if you take the slope of Line A $\left(-\frac{1}{2}\right)$ and flip and negate it, you will get the slope of Line B $\left(\frac{2}{1}\right)$.

Definition 3.4.7 Perpendicular lines are two lines that meet or intersect each other at a right angle. The slopes of two perpendicular lines are *negative reciprocals* of each other (given that the slope exists!). $\Diamond$

Activity 3.4.8 Suppose you have the function,

$$f(x) = 3x + 5$$

(a) What is the slope of $f(x)$?

A. $-\frac{1}{3}$

B. 3

C. 5

D. $-\frac{1}{5}$

(b) Applying Definition 3.4.7, what would the slope of a line perpendicular to $f(x)$ be?

A. $-\frac{1}{3}$

B. 3

C. 5

D. $-\frac{1}{5}$

(c) Find an equation of the line perpendicular to $f(x)$ that passes through the point $(3, 6)$.

Activity 3.4.9 For each pair of lines, determine if they are parallel, perpendicular, or neither.

(a)

$$f(x) = -3x + 4$$
$$g(x) = 5 - 3x$$

(b)

$$f(x) = 2x - 5$$
$$g(x) = 6x - 5$$

(c)

$$f(x) = 6x - 5$$
$$g(x) = \frac{1}{6}x + 8$$

(d)

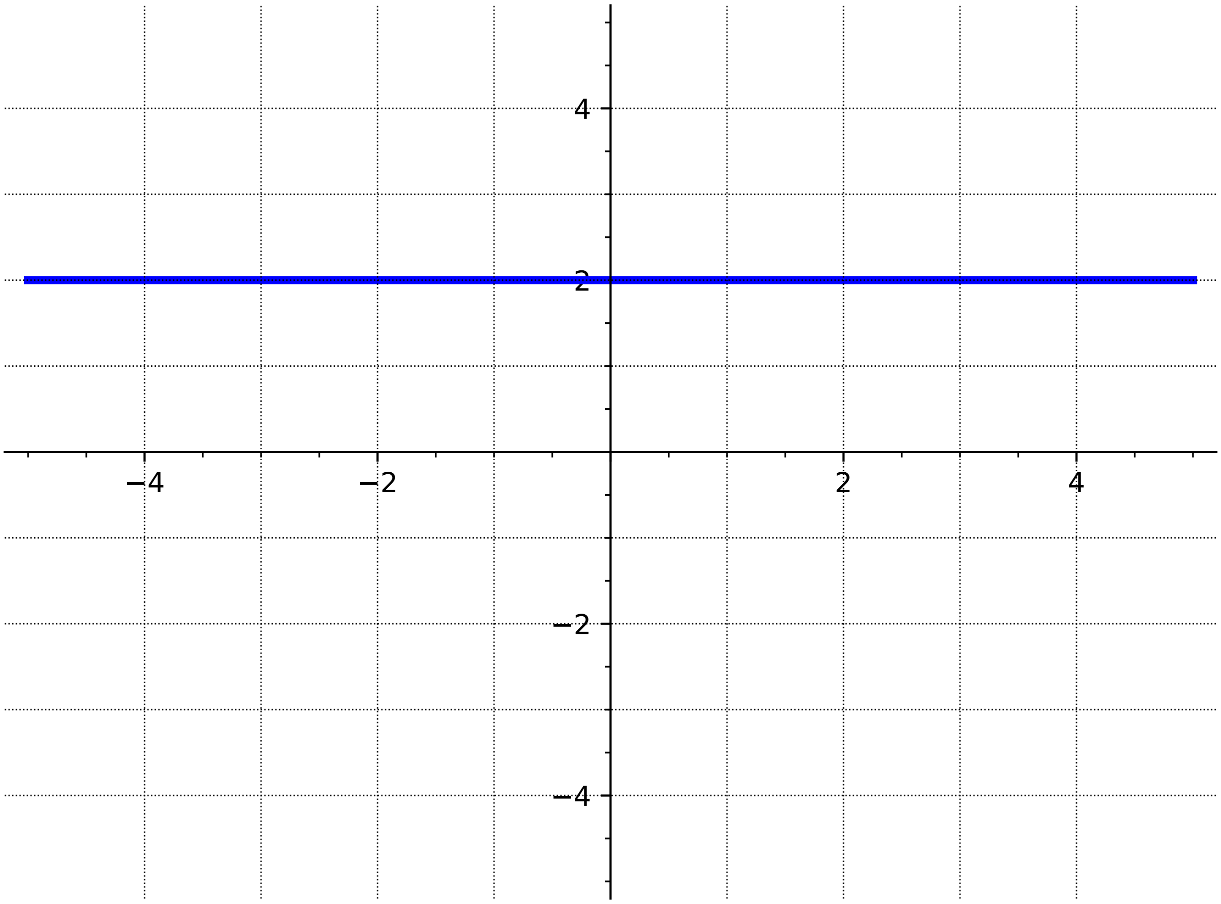
$$f(x) = \frac{4}{5}x + 3$$
$$g(x) = -\frac{5}{4}x - 1$$

Activity 3.4.10 Consider the linear equation, $f(x) = -\frac{2}{3}x - 4$ and the point A: $(-6, 4)$.

(a) Find an equation of the line that is parallel to $f(x)$ and passes through the point A.

(b) Find an equation of the line that is perpendicular to $f(x)$ and passes through the point A.

Activity 3.4.11 Consider the line, $y = 2$, as shown in the graph below.



(a) What is the slope of the line $y = 2$?

A. undefined

B. 0

C. 1

D. $-\frac{1}{2}$

(b) What is the slope of a line that is parallel to $y = 2$?

A. undefined

B. 0

C. 1

D. $-\frac{1}{2}$

(c) Find an equation of the line that is parallel to $y = 2$ and passes through the point $(-1, -4)$.

(d) What is the slope of a line that is perpendicular to $y = 2$?

A. undefined

B. 0

C. 1

D. $-\frac{1}{2}$

(e) Find an equation of the line that is perpendicular to $y = 2$ and passes through the point $(-1, 2)$.

3.5 Linear Models and Meanings (LF5)

Objectives

- Build linear models from verbal descriptions, and use the models to establish conclusions, including by contextualizing the meaning of slope and intercept parameters.

Remark 3.5.1 We begin by revisiting Activity 2.2.14 from Section 2.2.

Activity 3.5.2 Ellie has \$13 in her piggy bank, and she gets an additional \$1.50 each week for her allowance. Assuming she does not spend any money, the total amount of allowance, $A(w)$, she has after w weeks can be modeled by the function

$$A(w) = 13 + 1.50w.$$

(a) How much money will be in her piggy bank after 5 weeks?

(b) After how many weeks will she have \$40 in her piggy bank?

Remark 3.5.3 The function in the previous activity is an example of a **linear model**. A linear model is a linear function that describes, or models, a real-life application. In this section we will build and use linear models.

Activity 3.5.4 Jack bought a package containing 40 cookies. Each day he takes two in his lunch to work.

(a) How many cookies are left in the package after 3 days?

A. 46

B. 42

C. 38

D. 36

E. 34

(b) Fill out the following table to represent the number of cookies left in the package after the given number of days.

number of days	number of cookies left
0	
2	
5	
10	
20	

(c) What is the y-intercept? Explain what it represents in the context of the problem.

(d) What is the rate of change? Explain what it represents in the context of the problem.

(e) Write a linear function to model the situation. Let d represent the number of days elapsed and $C(d)$ represent the number of cookies in the package. (Hint: Use the previous two questions to help!)

(f) Find $C(6)$. Explain what that means in the context of the problem.

(g) How many days will it take to empty the package? What does this correspond to on the graph?

Activity 3.5.5 Daisy's Doughnut Shop sells delicious doughnuts. Each month, they incur a fixed cost of \$2000 for rent, insurance, and other expenses. Then, for each doughnut they produce, it costs them an additional \$0.25.

(a) In January, Daisy's Doughnut Shop produced 1000 doughnuts. What was their total monthy cost to run the shop?

A. \$2000.25

B. \$2002.50

C. \$2025.00

D. \$2250.00

E. \$4500.00

(b) Fill out the following table to represent the cost for producing various amounts of doughnuts.

number of doughnuts	**cost**
0	
500	
1000	
1500	
2000	

(c) What is the y-intercept? Explain what it represents in the context of the problem.

(d) What is the rate of change? Explain what it represents in the context of the problem.

(e) Write a linear function to model the situation. Let x represent the number of doughnuts produced and $C(x)$ represent the total cost. (Hint: Use the previous two questions to help!)

(f) Find $C(1300)$. Explain what that means in the context of the problem.

(g) Find the x-intercept. Explain what it means in the context of the problem.

Activity 3.5.6 A taxi costs \$5.00 up front, and then charges \$0.73 per mile traveled.

(a) Write a linear function to model this situation.

(b) How much will it cost for a 13 mile taxi ride?

(c) If the taxi ride cost \$11.06, how many miles did it travel?

Activity 3.5.7 When reporting the weather, temperature is given in degrees Fahrenheit (F) or degrees Celsius (C). The two scales are related linearly, which means we can find a linear model to describe their relationship. This model lets us convert between the two scales.

(a) Water freezes at $0°C$ and $32°F$. Water boils at $100°C$ and $212°F$. Use this information to write two ordered pairs.

Hint. Choose Celsius to be your input value, and Fahrenheit to be the output value.

(b) Use the two points to write a linear model for this situation. Use C and F as your variables.

(c) If the temperature outside is $25°C$, what is the temperature in Fahrenheit?

(d) If the temperature outside is $50°F$, what is the temperature in Celsius?

(e) What temperature value is the same in Fahrenheit as it is in Celsius?